A Functional Interpretation of Quantum Theory

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Abstract—In this paper a functional interpretation of quantum theory (QT) with emphasis on quantum field theory (QFT) is proposed. Besides the usual statements on relations between a functions initial state and final state, a functional interpretation also contains a description of the dynamic evolution of the function. That is, it describes how things function. The proposed functional interpretation of QT/QFT has been developed in the context of the author's work towards a computer model of QT with the goal of supporting the largest possible scope of QT concepts. In the course of this work, the author encountered a number of problems inherent in the translation of quantum physics into a computer program. He came to the conclusion that the goal of supporting the major QT concepts can only be satisfied, if the present model of QT is supplemented by a "functional interpretation" of QT/QFT. The paper describes a proposal for that.

Keywords—Computability, Foundation of Quantum Mechanics, Measurement Problem, Models of Physics.

I. INTRODUCTION

A Functional description of a given function shows explicitly a sequence of steps for the computation of the function results. For a user of the function this is often more information than is needed. However, for a complete understanding of a subject this is often requested.
Example: Sort function
A possible abstract (non-functional) definition of a Sort function would be
$Sort(\{x_1, x_2, ..., x_n\}) := \{x_{i1}, x_{i2}, ..., x_{in}\}$ with $x_{i1} \leq x_{i2} \leq ... \leq x_{in}$.
A possible functional description of a given Sort function would describe an algorithm, such as the following:

```
$Sort(\{x_1, x_2, ..., x_n\}) :=$ { For i = 1 to n do the following {
  as long as not $x_1 \leq x_2 \leq ... \leq x_i$
{ shift element $x_i$ to position j such that in the resulting set
$\{x'_1, ..., x'_j, ..., x'_n\}$ $x'_1 \leq x'_2 \leq ... \leq x'_j$.
} } }
```

This functional description of the Sort function describes, of course, just one of many alternative sort algorithms. For an abstract definition of the Sort function it provides unnecessary and possibly confusing details. However, for concrete subjects, for example a concrete realization of a Sort computer program, this additional detail may be of interest.
Typical constituents of a functional description are process steps, together with methods indicating how the process steps relate to each other (e.g. sequential, parallel, iterations, alternatives) and specifications of flow control (e.g. deciding on alternatives).

A *functional interpretation* of a theory adds to a given theory a (possible) interpretation in terms of process steps for the dynamic evolution of the system described by the theory.

The existing descriptions of QT, including QFT, describe in a more-or-less axiomatic fashion what can be expected at times when measurements are taken. What happens in between tends not to be dealt with in any detail.

Opinions differ among physicists about the need for a description of these in-between states, which in this paper the author terms a functional interpretation of QT.

1) Some do not view this as a deficiency at all. Why care about the intermediate steps as long as I understand what the result is.
2) Others assume that the lack of a functional interpretation is an inherent feature of QT. In the same way as we cannot measure position and momentum with arbitrary precision, or can't identify hidden variables as the source of the (seeming) nondeterminism, it may not be possible to describe what happens between time t0 and t1.
3) Others admit to feeling uneasy about not having a functional interpretation of QT, but consider this as a consequence of our (as yet) limited understanding of Nature. For example, R.Feynman writes in [3], page 82: "I have pointed out these things because the more you see how strangely Nature behaves, the harder it is to make a model that explains how even the simplest phenomena actually work. So theoretical physics has given up on that."
   Explaining "how phenomena actually work" could be understood as a functional interpretation.
   In [6], page 185, R.Penrose writes "present-day quantum mechanics is a *provisional* theory" because the non-unitary type of wave function evolution (associated with a measurement) is not yet understood.
4) Some criticize QT, or at least consider QT as being incomplete. A.Einstein is seen in this category, although he never complained about a missing functional interpretation of QT. However, his concern about the nature of (QT-) reality and the completeness of QT might have been dispelled if a functional interpretation of QT had existed.

The author disagrees with (1) and believes that for every subject of physics a functional interpretation must be feasible. With all fields of physics, except QT, functional interpretations, although not explicitly described, can easily be constructed.
With respect to (2) the author believes that this somewhat positivistic point of view has to often been taken to hide our incomplete understanding of QT.
Thus, he feels a functional interpretation of QT is justified .

The author first recognized the need for a functional interpretation of QT/QFT when he attempted to develop a comprehensive computer model of QT/QFT that supports the simulation of the majority of the (Gedanken-) experiments explaining the key concepts of QT. He soon had to realize that some of the key principles of QT are formulated in a way which does not allow their translation into a computer program, nor to precise mathematics. This holds true for some of the most important QT principles or statements:

1) The uncertainty principle
2) The principle of complementarity
3) The rule which defines when particle waves are in superposition and when superposition aborts
4) Statements on the measurement process

Fortunately, not all these principles are required to explain or predict the outcome of QT experiments. The principle of complementarity is not mandatory, and only a "lean" [1] version of the uncertainty principle is needed. However, items 3. and 4. are needed in some form. Looking at ways to express these rules in a computable form, it was found that in both cases the evolution of the wave function is the underlying subject. A model of the evolution of the wave functions is essential for a model of the evolution of a total QT system and for a functional interpretation of QT.

Thus, while within classical areas of physics, the creation of a functional interpretation (for specific processes) is the relatively easy exercise of mapping the largely axiomatic theory to a dynamic process model, with QT the task is impeded by the fact that some of its key principles cannot be mapped to precise mathematics or to process steps.

II. KEY FEATURES OF THE PROPOSED FUNCTIONAL INTERPRETATION

As described above, the provision of a functional interpretation of QT requires more than a translation from one kind of model (e.g. axiomatic model) to another kind of model. With some areas the existing theory needs to be modified and/or extended to make it suitable for a functional interpretation. Thereby, compatibility with standard QT/QFT has to be preserved to the largest possible extend. The following are of primary importance for the QT functional interpretation.

1) Criteria for Deciding when Interactions Imply a Collapse of the Wave Function: A functional interpretation of the measurement process requires, first at all, criteria for the distinction when an interaction implies a collapse of the wave function and when it does not. The criteria must be expressed in the terms common to the functional interpretation.

2) Coarse Graining of QT attributes: In [5], page 12, G. 't Hooft writes "Often, authors forget to mention the first, very important, step in this logical procedure: replace the classical procedure one wishes to quantize by a strictly finite theory. Assuming that physical structures smaller than a certain size will not be important for our considerations, we replace the continuum of three-dimensional space by a discrete but dense lattice of points. ".

The QT functional interpretation assumes discrete and coarse grain attributes not only for three-dimensional space, but for most other entities where standard QFT assumes differentiable attributes. This applies to the spatial extension of particle waves and to their momentum. Also, the wave function is structured into a discrete set of alternative paths.

Of course, the graining has to be kept fine enough to prevent significant deviations from the predictions obtained with standard QFT.

3) Handling of Non-Determinism - the Transition from Possibilities to Facts: This QT functional interpretation has to demonstrate the evolution of the wave function to generate probability amplitudes in accordance with the predictions of QFT. However, it does not end with the determination of probability amplitudes, but includes a model for the realization of the predictions represented by the probability amplitude. This is called "the transition from possibilities to facts". With standard QT, the transition from possibilities to facts is a non-deterministic process step which occurs exclusively with measurements.

One of the key features of the functional interpretation is that the transition from possibilities to facts is not exclusively tied to measurements. This means (a) that the measurement process loses its cumbersome special role, but it also means (b) that the progression of the wave function is no longer restricted to applying unitary operators to it, but non-deterministic functions may already apply to the ("normal") wave function progression.

The goal of the QT functional interpretation is that the (seeming) unitary progression of the wave function as assumed with standard QT remains as the approximation.

4) Particle Fluctuations Instead of Virtual Particles: With the perturbation (Feynman) approach, virtual particles are an essential concept for describing interactions among particles. The QT functional interpretation reinterprets the role of virtual particles; instead of the original QFT virtual particles, which always have a starting and an ending point, the QT functional interpretation assumes "*particle fluctuations*" which may result in transitions, but may also terminate without any effect. These particle fluctuations are assumed to actually happen (with a certain probability), while virtual particles are rather constructs for the calculation of the QFT scattering matrix.

5) Splitting of a Wave Function Collection into Multiple Paths: Splitting of a wave function into multiple paths is an essential ingredient of the perturbation approach to QFT (see [3]). The overall effect of the wave function progression is then determined by the superposition (via path integrals) of the multiple paths. With the QT functional interpretation, the splitting into multiple paths is applied to **collections** of particles having a common past history in the form of interactions among them. This allows modeling of entanglement.

III. BASIC MECHANISMS FOR THE EVOLUTION OF PARTICLE/WAVES

The functioning of the model can be described in terms of mechanisms.The most important basic mechanisms are described in this section. Since the QT functional interpretation does not distinguish between the particle and the (associated) wave, the term "particle/wave" will be used in the following.

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[1] The "lean" version of the uncertainty principle, is here understood to be the version where the uncertainty refers to the width of the wave function without reference to what *can be measured*. That version is expressed in precise mathematics and is translatable to a computer program.

In order to keep the description of the mechanisms reasonable compact and readable, but still somewhat formal, the detailed mechanisms are described in "pseudo-code". [2]

A. Mechanism for the Free Particle/Wave Propagation

The mechanism for the free particle/wave propagation is a simple translation of QFT propagator equations to a functional formulation. The only difference to the standard QFT propagator is, that the QT functional interpretation assumes the propagation to be interspersed by particle/wave-fluctuations. The particle/wave-fluctuations, however, become relevant only when they result in an interaction.

B. Mechanism for Interaction

There are two types of interactions between particle/waves: (1) interactions which only affect the attributes of the involved particle/waves, and (2) interactions which destroy the superposition among possible multiple paths of the wave function, resulting in "a collapse of the wave function". In this section the general mechanism which applies to both types of interactions is described. Type (2), interactions resulting in a collapse of the wave function is further described below in section IV.
With standard QFT, the probability amplitude for the occurrence of an interaction is in superposition with the amplitude for "no-interaction". This QT functional interpretation assumes that interactions are process steps that are actually happening (with a certain probability).

1) Particle/Wave Fluctuations: With standard QFT the treatment of an interaction includes virtual particles which are exchanged between the interacting real particles. While the proposed functional interpretation has to maintain the effects of virtual particle exchange, it uses a modified concept of particle/wave fluctuation (pw-fluctuation).

A pw-fluctuation can be thought of as temporary concentration and amplification of one or several particle/waves at a certain space-point. The following assumptions are essential for pw-fluctuations and their role in the QT functional interpretation:

- The position where the pw-fluctuation occurs can be anywhere within the space occupied by the involved particle/waves. The position is determined randomly as a function of the amplitudes of the involved particle/waves and of the fields involved.
- Only one pw-fluctuation may be active at a given point in time for a particle/wave.

```
generate-pw-fluctuation( ) :=  {
  FOR ( all pw-collections  pwc[i ]) {
    candidate-spacepoints = spacepoints with
      sum of amplitudes > 0;
  }
  fluct-position = RANDOM(candidate-spacepoints)
    weighted by candidate-spacepoints.amplitude;
  fluct-paths = all pathes of pw-collections
      with non-zero amplitude at fluct-position;
  ampl =
    sum of amplitudes of fluct-paths at fluct-position;
  pwset = all pw of fluct-paths;
  perform-pw-fluctuation (fluct-position, ampl,
        pwset);
}
```

The detailed shape of the pw-fluctuation is not important for the functional interpretation. The same is true for the dynamics of the pw-fluctuation evolution. For the functional interpretation, there is no need to make more detailed assumptions in this area beyond what is here described.

The immediate effect of a pw-fluctuation ("perform-pw-fluctuation()") is the temporary formation of an entity called pw-fluct-object. pw-fluct-object merges the information from the particle/waves (and the external fields) which caused the pw-fluctuation. When the temporary pw-fluct-object disappears again, the original particle/waves may be continued (or reinstalled), or a different set of particle/waves may appear. Accordingly, the long term effect of a pw-fluctuation can be either

1) nothing durable (this may be the case with the majority of the pw-fluctuations),
2) an interaction as described in the present section,
3) an interaction causing the collapse of the wave function (see section IV.),
4) a particle decay

2) Conditions for the Occurrence of Interactions: An interaction (cases 2. or 3. above) occurs when the pw-fluctuation was caused by two or more particle/waves.

```
perform-pw-fluctuation(pos,ampl,pwsetin) :=
{
  pw-fluct-object =
     genfluctobj(pwsetin,ampl,pos );
  pwsetout = pwseparation(pw-fluct-object);
  IF (pwsetout=pwsetin)
      performInteraction(pw-fluct-object,pwsetin);
  ELSE performInteractionWithCollapse
                (pw-fluct-object,pwsetout);
}
```

genfluctobj() and the separation of the pw-fluct-object into the pwsetout is not further detailed here. The process logic has to be in accordance with the rules of standard QFT defining the possible transitions of particle types (see, for example [7] and [2]). If the rules with respect to the separation of the pw-fluct-object allow for a variety of possible pwsetout (which is typically the case), the selection of pwsetout is done randomly. [3]

3) The Interaction Subprocess: In this section, only those interactions are considered where the exiting particle/wave set is identical to the ingoing set of particle/waves (the other cases are addressed in section IV.).

For pwsetout, the exiting particle/wave set, a large variety of attribute combinations is usually possible. These possible variations are not exclusive alternatives, but the interaction result contains all of them, although with different probability amplitudes. QFT provides the rules for the determination of

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[2] Pseudo-code is used in software development as a first step towards exact formal descriptions.

[3] This is a slight deviation from standard QFT which, however, will be difficult to test by experiments.

the possible resulting particle/wave combinations, including the associated probability amplitudes.

```
performInteraction(pw-fluct-object,pw-set) :=
{
 pw-collection-list =
 determine-list-of-attribute-combinations;
 FOR (all paths of pw-collection-list) {
  path.amp l= determine-amplitude
 (pw-fluct-object,path.attribute-combinations);
 }
}
```

The ”determine-amplitude()” function is as set out in the QFT rules.

4) Effects of an Interaction: For the QT functional interpretation, the most important effect of an interaction is the formation of a ”particle/wave-collection”.

The interaction results in changes to the attributes of the two involved particle/waves. In general, an interaction can have a variety of alternative outcomes (e.g. different momenta for the two involved particle/waves) with differing probability amplitudes. The decision on which alternative is chosen is made at the point of measurement. A further result of an interaction may be a correlation between the two particle/waves. As an example of such a correlation, an interaction may result in non-zero probability (-amplitude) for a range of momentum values for one of the exiting particle/waves. As soon as, due to a measurement, a specific momentum value is detected for one of the particle/waves, because of momentum conservation this will affect the possible momentum values of the other particle/wave involved in the interaction. The proposed QT functional interpretation handles the correlation by building the particle/wave-collection for the two correlated particle/waves. The particle/wave-collection consists of multiple paths, each path representing one of the alternative outcomes of a measurement.

paths	particle/wave	particle/wave	amplitude
path-1	$momentum^1_{pw1}$	$momentum^1_{pw2}$	ampl-1
path-2	$momentum^2_{pw1}$	$momentum^2_{pw2}$	ampl-2
...	...	...	...
path-n	$momentum^n_{pw1}$	$momentum^n_{pw2}$	ampl-n

Typically , the alternative paths of a particle/wave-collection differ in the momenta, positions, and spins associated to the particle/waves.

The mechanism for support of correlations among particles in form of pw-collections also applies in support of entanglement (see section VI.).

The QT functional interpretation applies the path-splitting mechanism only to interactions, while with standard QFT the normal (interaction-free) propagation of particle/waves may also be viewed as consisting of many paths.

The fact that the particle/wave-collection consists of a limited number of paths means that a coarse graining is also applied to the path splitting.

IV. Interactions Causing a Collapse of the Wave Function

With the interactions described so far (section III.B.), cases which result in a collapse of the wave function have not been considered. A ”collapse of the wave function” is understood as a phenomenon where a wave, which is a superposition of multiple paths, appears to be reduced to a single one.

For the QT functional interpretation the ”collapse of the wave function” is equated to the collapse of the pw-collection. This means, if one or both particle/waves involved in an interaction are part of pw-collections and these pw-collections have to be abandoned, this is an effect which is equivalent to the ”collapse of the wave function”.

The QT rule saying ”When it is possible to determine that a particle took a certain path, interference (of the paths) is lost” (see [4], page 1-13 for exact phrasing) is phrased in QT functional interpretation as follows:

”When it is possible to determine a correlation between the measured object (e.g. particle) and the measurement device (or the environment) and this correlation distinguishes possibly existing multiple paths within an ingoing pw-collection, that ingoing pw-collection is abandoned.” This rule is still not suited for translation to mathematics or to a computer program. Especially the part ”when it is possible to determine ” is not translatable to a formal language. Thus, as the next refinement step, a formulation which refers exclusively to the objects and terms of the physics theory is required. The functional interpretation of QT uses the following rule:

An ingoing pw-collection is always abandoned (i.e. collapses) whenever an outgoing pw-collection with a different structure has to be created.

The most obvious case where a different structure for the outgoing pw-collection is required is when the set of ingoing and outgoing particles differ. The QT functional interpretation assumes that this will always lead to the collapse of the ingoing particle/wave collections. For example, a particle decay or a pair annihilation always results in a collapse of the particle/wave collection. However, changes in the types of the involved particle/waves, as, for example with the process $e^- e^+ \to \mu^- \mu^+$, also result in collapse of the particle/wave collection.

Since the creation of a new pw-collection is the normal result of an interaction (see section III.B.4), and the contents of this outgoing pw-collection typically differs from a possible ingoing pw-collection, the collapse of the ingoing pw-collection is the normal case. Thus, the question is, under which circumstances the (ingoing) pw-collection does *not* collapse. The QT functional interpretation assumes that, when a particle/wave interacts with a bound system, such as an atom, under certain conditions the formation of a new pw-collection and the collapse of the ingoing pw-collection is omitted. Details on when the interaction with a bound system does not result in the collapse of the pw-collection will be worked out together with a more comprehensive treatment of bound systems. [4]

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[4] The treatment of bound systems is not yet sufficiently understood with QFT.

V. MODEL ON MEASUREMENT

Measurement plays a very important role with quantum theory. With standard QT the transition of possibilities to facts occurs exclusively with measurements. As described in section II., the QT functional interpretation assumes non-deterministic actions at different process steps. There is no explicit process (step) called measurement process, but measurement is explained in terms of general other process steps, such as ”Interactions causing a Collapse of the Wave Function” described above.

Besides the transition from possibilities to facts there are further concepts associated with measurement (i.e. with the collapse of the wave function). The most important of these is entanglement, which is tied to measurement.

VI. ENTANGLEMENT

As described in section III.B. above, the most important effect of an interaction is the formation of a particle/wave-collection. The particle/wave-collection supports correlations and entanglements between the particle/waves leaving the interaction.

The proposed QT functional interpretation handles entanglement by building the particle/wave-collection for the two entangled particle/waves. The particle/wave-collection encompasses multiple paths, each path representing one of the possible outcomes of a measurement. The measurement discards all paths except the one selected as the measurement result. As required by QT, there remains an uncertainty with the measurement of both entangled particles. Perfect correlation will only be found if the measurement is performed in terms of identical base vectors (for example, identical spin orientation).

The scheme described above for support of correlations and entanglements through the particle/wave-collection, also supports spin correlations. To what extent entanglements with ”perfect correlation” of spin can be achieved depends on the more detailed mechanism. Several alternatives are under investigation, but their discussion is outside the scope of this paper.

The entanglement concept of the QT functional interpretation maintains the strange non-locality of standard QT. However, the non-local effect applies to the elimination of alternatives, rather than to direct state changes.

VII. CONCLUSIONS

The major objective with the development of the QT functional interpretation described in this paper has been compatibility with standard QT/QFT. Nevertheless, there are areas where deviations from standard QFT are known and are intended. Some of these deviations are assumed to represent nothing more than differences in approximation. Beyond these, however, there remain deviations which are more than differences in precision. This is an area where experimentation is required to provide verification or falsification.

Prior to such experiments, it is possible to test compatibility with standard QFT by comparing computer simulations of standard QFT with computer simulations of the proposed QT functional interpretation. The computer program QTModel (see [1]) has been developed with this objective in mind. First results of QTModel simulation show agreement to the extent expected.

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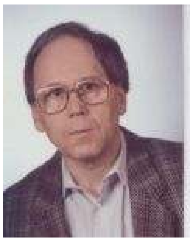

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