

On the Computability of Quantum Theory

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Abstract

In trying to develop a comprehensive computer simulation of Quantum Theory (QT), the author had to realize that, in addition to the areas of QT where computations are known to be difficult, other severe problems stem from major QT principles that cannot be translated into precise mathematics. In general, such problems do not necessarily indicate a deficiency in the theory. However, in this paper the author discusses to what extent these difficulties have to be considered as manifestations of other more severe problems in QT. The view of "not just computability problems" is supported by considerations of how the most severe computability problems relate to other major QT problems, such as the QT measurement problem. A direction for a possible solution is proposed.

Keywords: Computability, Foundation of Quantum Mechanics, Simulation of Quantum Physics.

Introduction

Like any theory of physics and most other areas of science, the value of a theory is largely measured by its ability to predict the outcome of experiments. Predictability is closely related to the computability of the rules and functions that make up the theory. Quantum theory (QT) can demonstrate enormous success in terms of predictability and computability. Surprising results have been predicted by the theory and subsequently confirmed by experiments.

Nevertheless, when the author tried to develop a comprehensive computer simulation of QT, he had to realize that in addition to the areas of QT where computations are known to be difficult; there are severe computability problems because some major QT principles cannot be translated into precise mathematics.

The computer model of QT whose development resulted in the detection of the described computability problems aims to support the simulation of a wide range of QT (Gedanken-) experiments. Many Gedanken-experiments described in the literature on QT are used to explain key QT concepts and principles, such as the uncertainty principle, particle/wave duality, and entanglement. Thus, for the computer model these principles had to be mapped to a computer program. For a comprehensive simulation of QT experiments it is also necessary to support interactions between particles (e.g., scatterings), which resulted in the need for support of quantum field theory (QFT) as well as the basic quantum mechanics.

Computability may be discussed from different perspectives. The mathematical theory of computability as introduced by A. Church, A. Turing et al. (see below) is not restricted to computations using a computer, although the theory can best be explained in terms of an idealized abstract computer or function calculus. Therefore, computability problems encountered with a theory may indicate problems of a more general nature than simply those related to the development of a suitable computer program. Section 2 addresses computability as introduced by A. Church, A. Turing, et al., i.e., computability as defined with mathematics.

Although the guiding perspective of this paper is the provision of computability required for a comprehensive QT computer simulation, computability problems encountered of a more fundamental nature have been found and are described in section 6. Current textbooks on QT describe QT principles that are not computable. It is not the case that these principles are non-computable because they are so complicated, nor because computers are not suited for their computation, but because they address non-physical terms and cannot be translated into precise mathematics. In section 8 a proposed solution for these QT computability problems is described under "A Functional Interpretation of QT".

Because the simulation of QFT is a major part of the QT computer simulation project, the computability problems with QFT are addressed in section 4. There are areas in QFT where the equations describing specific situations may be solved by clever mathematicians, but where computation by a computer program that has to support the general case is extremely difficult.

The Mathematical Definition of Computability

The mathematical theory on computability was developed first and foremost by A. Church, A. Turing, S. Kleene, E. Post, and K. Gödel (see [1],[2],[3]). The theory declares a function as "computable" if it can be programmed on an idealized "universal computer" such that the program terminates with the desired results for all valid input parameter combinations. A specific example of an extremely simple, universal computer is the so-called Turing machine. Other universal computers are computers that support, at least, the Turing machine capability. Computability in terms of a Turing machine is considered to be a rather general (i.e., computer independent) definition of computability. There is a tight relationship between functions that are computable/non-computable and predicates that are decidable/undecidable according to Gödel's theory. Equivalence of Turing-computability and

the more abstract concepts of λ -calculus and μ -recursive functions can also be shown. There are no known examples in which functions that have been identified as non-computable by a Turing-machine have been found computable by any special computer. There are many famous examples of problems that have been found to be undecidable (predicates) or non-computable (functions) in mathematics (e.g., the word problem of group theory or Hilbert's problem # 10) and in computer science (e.g., deciding on the equivalence of grammars).

Notice that the provision of a quantum computer would not extend the set of computable functions (or decidable predicates). Quantum computers (when available) can only compute the same class of functions as traditional computers, although with extremely improved performance.

In addition to the mathematical definition of Turing-computability and equivalent concepts, the computational complexity theory also provides a classification of types of computability with respect to the complexity of the problem. This classifies the computing effort required to solve a specific problem. For example, specific problems in which the effort required for the computation of a solution result grows as a polynomial with the size of the input parameter set have been identified. This class of problems is called "NP-complete problems". Similar to Turing-computability, the classification into "NP-complete" et al. does not depend on the computer being used, but applies to specific problems.

Impact: (on the provision of a comprehensive QT computer simulation): Except for the problem area discussed in section 6, the author has not encountered any non-computable functions or undecidable predicates with his computer model project. Theoretically, it is possible to define functions or predicates based on QT that are not computable or undecidable. However, physicists do not appear to have a need for such functions or predicates (except for the problems described in section 6).

The classification in terms of computing complexity such as NP-complete may identify certain computations as being unsuitable for a comprehensive QT computer simulation. However, such areas need to be discussed in the context of physics. (see section 4).

QT Example for Discussing Computability

A computer model of QT that aims to support the simulation of the major QT principles and the major QT (Gedanken-) experiments has to support the most famous QT experiment; the double-slit experiment. Unfortunately, however, support of the double-slit experiment leads to most of the computability problems described below. Therefore, the double-slit experiment is described here to the extent necessary for reference in later sections¹.

The double-slit experiment demonstrates one of the most important features of QT, the superposition of wave function paths. To also illustrate the destruction of the

¹The author assumes that the reader is familiar with the double slit experiment.

superposition, two variants of the double-slit experiment are usually discussed, (1) the experiment with photon sources near the slits and (2) the experiment without the photon sources. Figure 1 shows the photon sources, but is used for both cases.

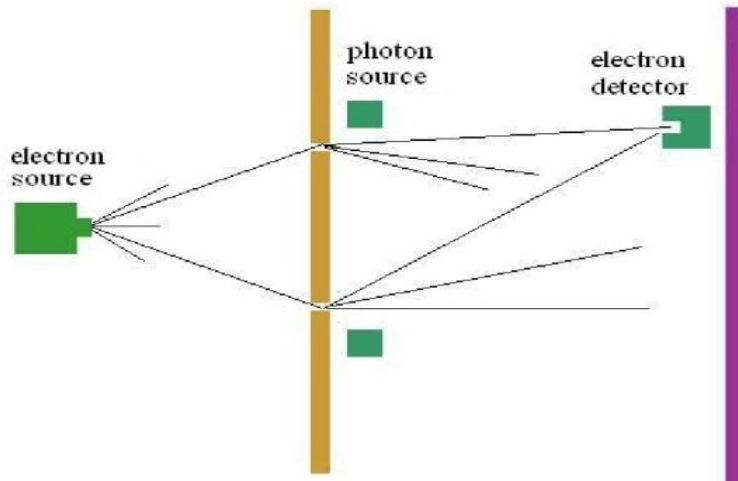


Figure 1: Double-Slit Experiment.

Double Slit Experiment without the Photon Sources Near the Slits

- The electron starts at the source with a spatial distribution such that the wave function propagates through both slits.
- The two paths partly reunite again behind the slits, producing the interference pattern at the electron detector.

Double Slit Experiment with Photon Sources at the Slits

In addition to the process described above, the electron wave passing the photon source may interact with a photon emitted by the photon source in such a way that the electrons wave function "collapses". Interference ceases.

QFT Areas With Poor Computability

Despite the impressive mathematical basis of QFT there are many areas of this theory where computations are very complex, and solutions for the equations can be demonstrated only for special cases.

For reasons described above, quantum field theory is the central part of the (comprehensive) simulation of QT. It was a surprise to the author that the availability of an extensive mathematical base for QFT does not automatically imply good computability in all areas. There are three main areas where computability is a problem:

Computation in Coordinate Space

Computations of probabilities, or cross sections for the results of QFT processes such

as scatterings or decays, may be performed in momentum space or in co-ordinate space. Computation in momentum space requires that the momenta for the initial and final states of the particles be specified. Likewise, computation in coordinate space requires that the positions of the particles be specified.

Studying textbooks on QFT (see for example [4], [5],[6], [7]) at first glance it appears as if computations in momentum space and in coordinate space are two alternatives on equal footing. The fact that computation in coordinate space is not dealt with in great detail in textbooks may be due to the larger applicability of computations in momentum space. After deeper study, it becomes clear that computations in coordinate space can be very difficult and may lead to equations for which solutions do not exist at all or can only be found with special cases.

While it may be said that computations in coordinate space are poorly described and understood, this is even more true for the general case, computations where momenta as well as positions are given with specific distributions. QT requires that the momentum and the position assigned to a particle have a certain distribution (for example, a specific Gauss-distribution) represented by the wave function $\varphi(\cdot)$. With a wave function $\varphi(p)$ for the momentum p , and $\varphi(x)$ for the position x , a distribution width $\Delta_d(p)$ and $\Delta_d(x)$ can be defined. QT states that $\varphi(p)$ and $\varphi(x)$ are correlated such that one can be determined from the other via Fourier-transformation

$$\varphi(p) = \text{FourierTransl}(\varphi(x)) \text{ and vice versa. Furthermore}$$

$$\Delta_d(p) * \Delta_d(x) \geq h/2. \quad ^{2,3}$$

QFT computation in momentum space means that the momenta have exact values ($\Delta_d(p) = 0$), which means the possible position values are completely arbitrary. For computations in coordinate space the positions are exactly known and the momenta are completely undefined. Directions for computation for the general case with $\Delta_d(p) > 0$; $\Delta_d(x) > 0$ could not be found in the literature.

In addition to the mathematical difficulties associated with the treatment of computations in coordinate space and of the general case, the main reason for the lack of computational direction is most likely the fact that computations in momentum space are sufficient for the purposes of most QFT physicists.

Impact: For a computer model of QT/QFT intended to support the largest possible scope of QT/QFT, it is desirable that the positions of particles and of measurement devices (for example the positions of the particle sources and the slits in the double slit experiment) can be configured by the user. Exclusion of computations in coordinate space and of computations for the general case (i.e., $\Delta_d(p) > 0$; $\Delta_d(x) > 0$), would also exclude simulation of many Gedanken-experiments.

² This may be considered as a variant of Heisenbergs uncertainty relation $\Delta p * \Delta x \geq h/2$. See also section 6.3

³ assuming the appropriate definition of Δ_d .

Computation of Higher Orders of Perturbation

The main approach for the computation of the probabilities for the results of QFT processes (e.g., scatterings, decays) is the so-called perturbation approach developed by Feynman and Dyson. The perturbation orders resulting in improved precisions of the computation results are characterized by an increased number of intermediate points (= vertexes) on the paths from the initial state towards the final state. The computation for the lowest order of perturbation is straight forward. Higher orders of perturbation can be very difficult. In terms of the related Feynman diagrams, the higher orders of perturbation often result in loops. Often the related integrals are diverging, leading to results that are physically meaningless. With standard QFT, the diverging integrals are tackled by very sophisticated techniques called regularization and renormalization. These techniques are not suited for implementation by a computer program which aims at supporting the general cases.

Impact: For the provision of a comprehensive QT computer simulation, exclusion of situations requiring higher perturbation orders is not considered acceptable by the author. At the least, cases that lead to problems should be recognized by the computer program and signaled to the user.

Computations for Bound Systems

Predictions for the behavior of bound systems, such as an atom, a nucleus, or a hadron, can be computed with QFT for special situations only, and only with high effort. QFT includes some theory and considerations on bound systems (see [4], [8], [7]), but this does not include a complete and consistent description of the total system in terms of QFT constructs such as Feynman diagrams, etc.. In [4] S.Weinberg writes "It must be said that the theory of relativistic effects and radiative corrections in bound states is not yet in entirely satisfactory shape".

Impact: Support of simulation of bound systems would enormously increase the value of a QT computer simulation. Conversely, the omission of support of bound systems injures the completeness of a (claimed) comprehensive QT computer simulation.

Non-Determinism and Chaotic Behavior

Although non-deterministic and chaotic processes are difficult to compute, these types of functions do not represent a problem for a comprehensive QT computer simulation (albeit for different reasons).

Impact: Non-determinism: The result of QT computer simulation will be probabilities. The computation of these probabilities is a deterministic computation.

Chaotic processes: As described in [9], chaotic behavior can seldom be found with QT. If at all, it is mainly anticipated at the boundaries where QT systems turn into classical systems (for example at the uppermost electron orbits in an atom). Non-support of such situations does not significantly reduce the value of an otherwise

comprehensive QT computer simulation.

QT Principles That Cannot be Translated to a Computer Program

Although QT is supported by an extensive and exacting mathematical framework, and many important discoveries (e.g., of new particle types) were the result of mathematical reasoning, there are a few places which are formulated in plain natural language and which do not seem to be translatable to a computer program. The occurrence of these "non-translatable" principles looks like the occurrence of non-computable functions in other areas of science (for example, the non-computable functions and undecidable predicates known from mathematics and computer science). However, the problem with these natural language formulated QT principles is even more severe. The real problem is that these principles do not seem to be translatable to mathematics or to any other formal language.

When Do Probabilities As Opposed To Probability Amplitudes Need To Be Added?

One of the basic features of QT is that the probability of an event for which multiple alternative paths are possible is a function of the superposition of the wave functions of the multiple paths. Superposition means summing up the probability amplitudes of the wave functions. In cases where there is no superposition, the probabilities for the multiple paths have to be added, which means the classical (i.e., non-QT) behavior is exhibited.

The rule for deciding when probabilities as opposed to probability amplitudes have to be added is one of the basic laws of QT. In [10] Feynman phrases it as follows:

"When an event can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately. There is interference:

$$\begin{aligned}\phi &= \phi_1 + \phi_2 \\ P &= |\phi_1 + \phi_2|^2\end{aligned}$$

If an experiment is performed which is capable of determining whether one or another alternative is taken, the probability of the event is the sum of the probabilities for each alternative. The interference is lost.

$$P = P_1 + P_2 \text{ ."} \text{ (end of citation)}$$

This rule has to be implemented by any computer simulation that aims at supporting a reasonable set of the classical QT Gedanken-experiments. The double slit experiment (see section 3) is usually used to illustrate the rule. Without a photon source near a slit, interference occurs. Inclusion of a photon source aborts the interference.

The author's attempt to incorporate this rule into a QT computer model (see [11]) showed that this was not possible.

The problem is that a condition such as "an-experiment-is-performed-which-is-capable-of-determining-whether-one-or-another-alternative-is-taken" cannot be understood as a trivial self-explanatory predicate, nor can it be reasonably defined in terms of any other mathematical construct. It may be possible to find an alternative, somewhat more concrete phrasing of the above rule such as "The probability amplitudes have to be added (i.e., are in superposition) unless through a type of "measurement-like interaction" it is possible to determine that a particular path is taken.". This phrasing still requires that the term "measurement-like interaction" be more precise to attain computability. However, when a situation may be considered as representing exactly a measurement-like interaction is one of the open questions of the unresolved measurement problem (see below).

Impact: As described above, the formation and destruction of interference is part of many QT (Gedanken-) experiments. Therefore, a computable rule which determines the cancellation of interference is mandatory for the provision of a comprehensive computer simulation of QT.

The Measurement Problem - What Happens When a Measurement Takes Place?

The so-called "measurement problem" was identified as one of the major open issues of QT soon after the formulation of quantum mechanics. There is no agreed-upon theory describing what causes a measurement, and what exactly happens when a measurement takes place. Various so-called interpretations of QT (see section 7.2) are related to this problem.

Impact: At first glance it is not clear why the unsolved measurement problem should impede a computer simulation of QT/QFT. However, on reflection, at least two considerations come to mind:

- A QT computer simulation would be more complete if instead of ending at the point of measurement, the measurement process were included.
- In addition to measurements representing the end of a QT/QFT experiment, there are, according to the QT interpretation favored by the author, many measurement-like types of interactions which have to be supported by a comprehensive QT computer simulation. An example when the interaction leads to the superposition being aborted is the one described in section 6.1.

The Uncertainty Principle

Heisenberg's famous uncertainty principle occurs with many Gedanken-experiments for one of the following reasons:

- The uncertainty principle is used to argue that because of this principle, some other principle (for example the ones addressed in section 6.1) holds true (in the specific case).
- Gedanken-experiments are often used to illustrate the uncertainty principle itself.

Differing from the computability problem described in section 6.1 above, the uncertainty principle has a very precise (and simple) mathematical formulation

$$(3) \Delta p * \Delta x \geq h/2 .$$

However, in textbooks on QT, two versions of the uncertainty principle, here called the lean version and the extended version, can be found.

- Lean version: Δp and Δx are associated with the width of the wave functions $\varphi(p)$ and $\varphi(x)$. As described in section 4.1, the distribution of the momentum is a function of position distribution, and the relation $\Delta p * \Delta x \geq h/2$ can be derived from this.
- Extended version: Often the lean version is generalized to say something like: "If the position is known with accuracy (or certainty) Δx , the momentum can only be known with limited accuracy Δp , such that $\Delta p * \Delta x \geq h/2$ ".

The computation for the lean version is trivial. Section 4.1 contains some comments on this. The computation for the extended version faces similar problems as with the principle described in section 6.1. A condition like "can-only-be-known with..." can hardly be mapped to any precise mathematical constructs. For the same reasons it cannot be mapped to a computer program.

Impact: As described above, computation of the lean version is trivial. It is a consequence of some basic equations of QT and will therefore be supported implicitly.

Explicit support (i.e., support which can be controlled by the user) of the extended version of the uncertainty principle by a QT computer simulation would be difficult (or impossible) for the reasons described above. Requirement for an explicit support is hardly imaginable. The uncertainty principle (at least the extended version) should be understood as a conjecture which is satisfied (or not) implicitly when the basic laws of QT are implemented.

Principle of Complementarity

The principle of complementarity is an important part of the Copenhagen Interpretation of QT. In the current literature on QT, this principle is still frequently used to explain certain Gedanken-experiments.

Whatever an exact agreed-upon formulation of the principle of complementarity may be, it is hardly imaginable that it can be translated to precise mathematics or to a computer program.

Impact: As with the uncertainty principle, the requirement for explicit support of the principle of complementarity is hard to imagine.

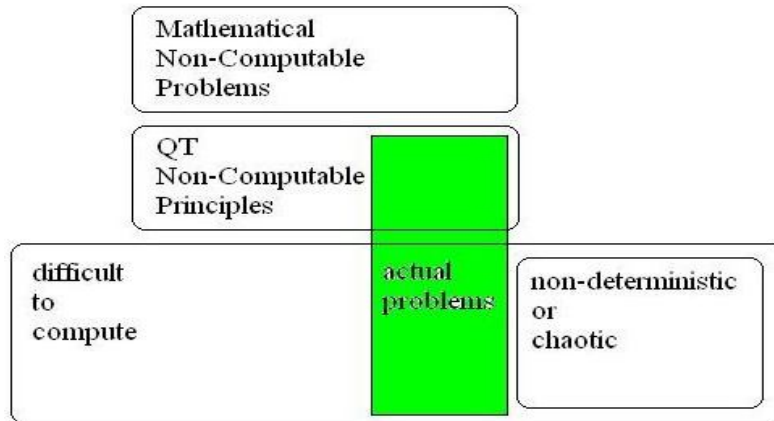


Figure 2. QT Problems Associated with a Comprehensive QT Computer Simulation

What is the Problem?

Figure 2 summarizes the various types of potential and actual computation problems discussed in the preceding sections (the sizes of the boxes have no relevance).

Potential problems:

- Mathematical Non-Computable Problems (section 2)
- QT Non-Computable Principles (section 6)
- Difficult to Compute (section 4)
- Non-Deterministic or Chaotic (section 5)

The problems that actually impede the development of a comprehensive QT computer simulation are shown in green.

The following discussion addresses the most severe computability problems and to what extent they represent more than "just" computability problems.

QT Computation Problems

Physicists concerned with QT are well aware that computation of QT is difficult, and that resolution of this problem remains an ongoing task. That the existing computability problems impede the provision of a comprehensive QT computer simulation of QT is a logical consequence, and should not be a surprise. That some of the major QT principles described in section 6 are not computable, to the authors knowledge, has not yet been precisely stated anywhere. Nevertheless, many physicists seem to have foreseen this difficulty. That the existence of these QT computation problems adversely affect the feasibility of a comprehensive (i.e., supporting all the major QT principles) QT computer simulation may be regretted by some physicists, but considered unavoidable.

The author's position is that the problem is more severe than the simple impediment of a comprehensive computer simulation. The fact that some of the major principles of a theory are (a) (less severely) not computable, (b)(more severely) not

translatable to precise mathematics, and (c) (more severely) are not falsifiable should not be excused by the strange nature of QT.

Before discussing this claim in relation to other QT problems, let us further narrow down the problem. The most severe computability problems discussed in the preceding sections are those described in section 6 "QT principles that cannot be translated to a computer program". Among the principles discussed, some are not seen as inhibitors for a comprehensive QT computer simulation, because their explicit support is not required for demonstrating QT. The problems which cannot be ignored are the ones related to the "collapse of the wave function" described in 6.1 "When Are Probabilities As Opposed To Probability Amplitudes Need To Be Added?" and 6.2 "The Measurement Problem".

Relation to Other QT Problems

The Measurement Problem In the context of the present paper, the measurement problem can be expressed by two questions:

- What constitutes a measurement ?
- What happens during a measurement ?

In terms of QT concepts, the problem may also be phrased as:

How can the collapse of the wave function (i.e., the non-unitary type of wave function evolution) be explained, or are there satisfactory alternatives to the (apparent) collapse? More precise descriptions of the measurement problem can be found in the literature for example, [12], [13], [14]). Many proposals have been made to answer these questions (a collection of proposals and discussions is contained in [14]). Additionally, "interpretations of QT" (see next section) have been proposed in an attempt to provide answers to this problem. These proposals do not seem to have convinced the QT experts. In [15] R. Penrose writes with respect to the non-unitary type of wave function evolution (associated with a measurement) "present-day quantum mechanics is a provisional theory".

As described below, the author believes that the so-called measurement problem is mainly a quest for more details with respect to the measurement process and that such process-oriented details can only be provided by a functional description described below.

The most severe computability problems described in this paper (the ones described in Section 6.1 and 6.2) are related to measurement and measurement-like interactions. The author claims that a satisfactory solution to the measurement problem will also remove these computability problems and vice versa.

Interpretations of Quantum Theory Mainly as reactions to the measurement problem, various so-called interpretations of quantum theory have been proposed. Some of the most famous are:

- Copenhagen interpretation (see [13])
- Hidden-Variables interpretation (see [13])
- Many Worlds interpretation (see [13])
- Many Minds interpretation (see [16])

- Transactional Interpretation (see [17])

Detailed treatment of these interpretations of QT is outside the scope of this paper (but see [18], [16], [15], [13]).

The author sees two problems related to these "interpretations" of QT:

- Their purpose is unclear. The term "interpretation" suggests that it is an add-on to the existing QT, which may provide additional understanding, but which is not really mandatory for the completeness of the theory.
- The requirements with respect to the scientific standards are unclear. Mainly as a result of the first problem, the creators of some of the interpretations seem to feel that the scientific standards which are generally accepted with theories of physics may be disregarded with respect to these interpretations. Indeed, some interpretations are even woolly, or refer to terms outside the scope of QT. That these interpretations are not falsifiable seems to be widely accepted in the same way as the fact that the rules are not computable.

As mentioned above the QT interpretations are primarily attempts to solve the measurement problem, and the measurement problem is primarily a quest for more details of the measurement process. QT interpretations which only propose a direction for the missing details, or which leave the scope of proper physics can hardly provide the missing details.

A satisfactory solution to the measurement problem should alleviate the need for an "interpretation" of QT, and as mentioned above, remove the most severe computability problems.

The Lack of a Functional Description of QT In [19] R. Feynman writes "I have pointed out these things because the more you see how strangely Nature behaves, the harder it is to make a model that explains how even the simplest phenomena actually work. So theoretical physics has given up on that."

Explaining "how phenomena actually work" is called in this paper "a functional description". A functional description of a given function shows explicitly a sequence of steps for the computation of the function results. For a user of the function this is often more information than is needed. However, for a complete understanding of a subject, this is often required.

Example: Sort function

A possible abstract (non-functional) definition of a Sort function would be

$$\begin{aligned} \text{Sort} (\{x_1, x_2, \dots, x_n\}) : \\ = \{x'_1, x'_2, \dots, x'_n\} \text{ with } x'_1 \leq x'_2 \leq \dots \leq x'_n . \end{aligned}$$

A possible functional description of a given Sort function would describe an algorithm, such as the following:

$$\begin{aligned} \text{Sort} (\{x_1, x_2, \dots, x_n\}) : \\ = \{ \text{For } i = 1 \text{ to } n \text{ do the following } \{ \text{as long as not } x_1 \leq x_2 \leq \dots \leq x_i \\ \{ \text{shift element } x_i \text{ to position } j \text{ such that in the resulting set } \{x'_1, \dots, x'_j, \dots, x'_n\} \end{aligned}$$

$$x'1 \leq x'2 \leq \dots \leq x'j. \} \}$$

This functional description of the Sort function describes, of course, just one of many alternative sort algorithms. For an abstract definition of the Sort function it provides unnecessary and possibly confusing details. However, for concrete subjects, for example a concrete realization of a Sort computer program, this additional detail may be of interest.

Typical constituents of a functional description are process steps together with methods indicating how the process steps relate to each other (e.g., sequential, parallel, iterations, alternatives).

A functional interpretation of a theory adds to a given theory a (possible) interpretation in terms of process steps for the dynamic evolution of the system described by the theory.

The existing descriptions of QT, including QFT, describe in a more-or-less axiomatic fashion what can be expected at times when measurements are taken. What happens in-between tends not to be dealt with in any detail.

Opinions differ among physicists about the need for a description of these in-between states, which in this paper the author terms a functional description of QT.

- Some do not view this as a deficiency at all. Why care about the intermediate steps as long as I understand what the result is?
- Others assume that the lack of a functional interpretation is an inherent feature of QT. In the same way as we cannot measure position and momentum with arbitrary precision, or cannot identify hidden variables as the source of the (seeming) nondeterminism, it may not be possible to describe what happens between time t_0 and t_1 .
- Others admit to feeling uneasy about not having a functional interpretation of QT, but consider this as a consequence of our (as yet) limited understanding of Nature. For example, R. Feynmans' remark cited above (and in [19]) is interpreted in this way by the author.
- Some criticize QT, or at least consider QT as being incomplete. Einstein is seen in this category, although he never complained about a missing functional interpretation of QT. However, his concern about the nature of (QT-) reality and the completeness of QT might have been dispelled if a functional interpretation of QT had existed.
- The author disagrees with (1) and believes that for every area of physics a functional description must be feasible. With all fields of physics, except QT, functional descriptions, although not explicitly described, can easily be constructed⁴. With respect to (2) the author believes that this somewhat positivistic point of view has too often been allowed to obscure the fact that our understanding of QT is incomplete.

⁴The functional descriptions can often largely be derived from the differential equations of the theory.

Functional interpretations of a theory are inherently better computable than axiomatic descriptions. This also holds true for the QT computability problems described in this paper⁵. With the author's work towards a functional interpretation of QT (see [21]) he has concluded that (a) a functional interpretation of QT without a functional description of the measurement process will be insufficient, and vice versa (b) a satisfactory solution of the measurement problem requires a functional view of the theory.

Direction for a Functional Interpretation of QT

The functional interpretation proposed in [21] is based on QFT with the perturbation (Feynman) approach. It describes the evolution of wave functions (including possible interactions) in terms of process steps (i.e., functions). Key features of the proposed functional interpretation are

- Criteria for deciding when interactions result in a "collapse of the wave function"
- Coarse graining for attributes and path subdivision,
- Particle/wave fluctuations taking the role of virtual particles,
- Transition from probabilities to facts not tied to measurements, and
- Path collections in support of entanglement.

Conclusion

A number of computability problems that impede the provision of a comprehensive computer model of QT have been described. Most of the problem areas represent deficiencies with respect to the state of computability in QT/QFT. The theory, including the mathematical basis, is available. The available mathematical basis, however does not (yet) imply satisfactory computability in all areas.

In addition to these computability difficulties, there are also the more fundamental problems due to QT principles that cannot be mapped to a computer program. It is explained why this lack of computability is more disquieting than the computing difficulties and the non-computable or undecidable problems known in mathematics. The problem is not only that the principles cannot be translated into a computer program but also that they can neither be translated into mathematics nor into any alternative formal language. In section 7 it is described how these severe QT computability problems relate to some other major QT problems. It is concluded that the problem areas are intertwined. A direction for a solution is outlined under the name "functional interpretation of QT".

⁵The terms "functional description" and "computational description" are also used in [20] where also the advantages of functional descriptions are described.

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