

Article

Emerging Quantum Fields Embedded In The Emergence Of Spacetime.

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Academic Editor: name

Version June 11, 2018 submitted to Entropy

Abstract: Based on a local causal model of the dynamics of curved discrete spacetime, a causal model of quantum field theory in curved discrete spacetime is described. On the elementary level, space(-time) is assumed to consist of interconnected space points. Each space point is connected to a small discrete set of neighboring space points. Density distribution of the space points and the lengths of the space point connections depend on the distance from the gravitational sources. This leads to curved spacetime in accordance with general relativity. Dynamics of spacetime (i.e., the emergence of space and the propagation of space changes) dynamically assigns "in-connections" and "out-connections" to the affected space points. Emergence and propagation of quantum fields (including particles) are mapped to the emergence and propagation of space changes by utilizing identical paths of in/out-connections. Compatibility with standard quantum field theory (QFT) requests the adjustment of the QFT techniques (e.g., Feynman diagrams, Feynman rules, creation/annihilation operators), which typically apply to three in/out connections, to $n > 3$ in/out connections. In addition, QFT computation in position space has to be adapted to a curved discrete space-time.

Keywords: spacetime models, discrete spacetime, relativity theory, causal models, quantum field theory, spin networks, quantum loops

1. Introduction

The author's attempt to construct a local causal model of quantum theory (QT), including quantum field theory (QFT), soon resulted in the recognition that a causal model of the dynamics of QT/QFT should better be based on a causal model of the dynamics of spacetime. Thus, a causal model of the dynamics of spacetime has been developed with the major goals (1) as much as possible compatibility with general relativity theory (GRT), and (2) the model should match the main features of the evolving model of QT/QFT. The main features of the author's model of QT/QFT are

- the model has to be a causal model,
- if possible, the model should be a *local* causal model,
- discreteness of the basic parameters (time, space, propagation paths).

Not surprisingly, it turned out that a clear definition of these features/requirements, especially of a local causal model, is useful (not only for understanding the requirements, but also for the derivation of the implications). A semi-formal definition of a (local) causal model has been published in several articles from the author (see [1], [2] and [3]) and is also given in Section 2.

The construction of a causal model of spacetime dynamics started with the search for some existing theory or model which might be at least a starting point for the model to be developed. Causal dynamical triangulation (CDT, see [4], [5], [6]) and more abstractly the concepts of loop quantum gravity (see [7] and [8]) were identified to match the author's requirements and thinking. The further

35 model construction showed that, in order to come up with a local causal model according to the
 36 definitions given in Section 2, adaptations and refinements of the original CDT-based model appear
 37 appropriate. The adaptations and refinements concern basic GRT concepts such as (i) the elementary
 38 structure of space(-time), (ii) the representation of space(-time) curvature, and (iii) the relation between
 39 space and time. With GRT and special relativity theory (SRT), space and time are said to be integrated
 40 into spacetime. For the GRT-compatible model of spacetime dynamics, the integration of space and
 41 time remains, but with a different interpretation. The elementary structure of space(-time), including
 42 the space-time relationship is described in Section 3. The causal model of the spacetime dynamics is
 43 described in Section 4.

44 The major goal for the development of a causal model of spacetime dynamics (Sections 3 and
 45 4) was to develop a model of the spacetime elementary structure that constitutes a suitable base
 46 for both the causal model of spacetime dynamics and the causal model of QT/QFT. The proposed
 47 model satisfies this goal. The emergence and propagation of quantum fields (including particles)
 48 can be mapped to the emergence and propagation of space changes by utilizing identical paths of
 49 in/out-connections between space points. In Section 5, this main subject of the article is described.

50 2. Causal Models

51 The specification of a causal model of a theory of physics consists of (1) the specification of the
 52 system state, (2) the specification of the laws of physics that define the possible state transitions when
 53 applied to the system state, and (3) the assumption of a “physics engine.”

54 2.0.1. The physics engine

55 The physics engine represents the overall causal semantics of causal models. It acts upon the state
 56 of the physical system. The physics engine continuously determines new states in uniform time steps.
 57 For the formal definition of a causal model of a physical theory, a continuous repeated invocation of
 58 the physics engine is assumed to realize the progression of the state of the system.

```
59
60 physics engine (S, Δt) := {
61     DO UNTIL(nonContinueState(S)) {
62         S ← applyLawsOfPhysics(S, Δt);
63     }
64 }
```

65 2.0.2. The system state

66 The system state defines the components, objects and parameters of the theory of physics that can
 67 be referenced and manipulated by the causal model. In contrast to the physics engine, the structure
 68 and content of the system state are specific for the causal model that is being specified. Therefore, the
 69 following is only an example of a possible system state specification.

```
70
71 systemstate := {spacepoint...}
72 spacepoint := {x1, x2, x3, ψ}
73 ψ := {stateParameter1, ..., stateParametern}
```

74

75 2.0.3. The laws of physics

76 The refinement of the statement

77 $S \leftarrow \text{applyLawsOfPhysics}(S, \Delta t)$; defines how an "in" state s evolves into an "out" state s .

78 $L_1 := \text{IF } c_1(s) \text{ THEN } s \leftarrow f_1(s)$;

79 $L_2 := \text{IF } c_2(s) \text{ THEN } s \leftarrow f_2(s)$;

80 ...

81 $L_n := \text{IF } c_n(s) \text{ THEN } s \leftarrow f_n(s);$

82 The "in" conditions $c_i(s)$ specify the applicability of the state transition function $f_i(s)$ in basic formal
83 (e.g., mathematical) terms or refer to complex conditions that then have to be refined within the formal
84 definition.

85 The state transition function $f_i(s)$ specifies the update of the state s in basic formal (e.g.,
86 mathematical) terms or refers to complex functions that then have to be refined within the formal
87 definition.

88 The set of laws L_1, \dots, L_n has to be complete, consistent and reality conformal (see [9] for more details).

89 In addition to the above-described basic forms of specification of the laws of physics by $L_n :=$
90 $\text{IF } c_n(s) \text{ THEN } s \leftarrow f_n(s)$, other forms are also imaginable and sometimes used in this article. (This
91 article does not contain a proper definition of the used causal model specification language. The
92 language used is assumed to be largely self-explanatory.)

93 2.1. Requirements for causal models of spacetime

94 For causal models of spacetime, obviously, some notion of space and time must be supported.
95 Ideally, the treatment of space and time would be, as much as possible, compatible with special
96 relativity theory (SRT) and general relativity theory. However, the formally defined causal model
97 of Section 2 presupposes a certain structure of spacetime in which space and time are rigorously
98 separated. This disturbs the integrated view of space and time that is taught by GRT/SRT. In the
99 proposed model of spacetime dynamics, the integration of space and time is largely restored by the
100 specification of the relationships described in Section 3.1.

101 2.1.1. The representation of time in the causal model

102 In the causal model defined above, time is not, like space and other parameters, a system state
103 component, but it has a special role outside the system state. The overall purpose of the causal model
104 is seen in showing the progression of the system state in relation to the progression of time. This
105 relationship can best be described by assuming a uniform progression of the time. This leads to the
106 model (described above) where the time and the progression of time is built into the model in the form
107 of the physics engine. The physics engine progresses the system state in uniform time steps called
108 state update time intervals (SUTI).

109 In GRT and SRT, there are situations where the clock rate of a causal subsystem is predicted to
110 differ depending on the relative speed of movement or the position within a gravitational field. GRT
111 and SRT refer to this by the name "proper time". If, for a specific causal model of an area of physics
112 the differing proper times of causal subsystems are relevant and/or the internal processes within the
113 subsystems are included in the model, separate physics engines may be assigned to the subsystems
114 with different proper times. An example can be found in the causal model described in [3], where
115 separate physics engines are assigned to the "quantum objects".

116 If, however, the causal model describes an area of physics where the relationship between proper
117 times and other parameters is to be shown, it should be possible to show this with a single physics
118 engine and a uniform SUTI for the overall system. For the proposed causal model of spacetime
119 dynamics, the space-time relationship described in Section 3.1 enables a single physics engine and a
120 uniform SUTI.

121 2.1.2. Spatial causal model

122 A causal model of a theory of physics is called a *spatial* causal model if (1) the system state contains
123 a component that represents a space, and (2) all other components of the system state can be mapped
124 to the space. There exist many textbooks on physics (mostly in the context of relativity theory) and
125 mathematics that define the essential features of a "space". For the purpose of the present article, a
126 more detailed discussion is not required. For the purpose of this article and the subject locality, it is

127 sufficient to request that the space (assumed with a spatial model) supports the notions of position,
128 coordinates, distance, and neighborhood.

129 A special type of spatial causal model that has been increasingly addressed in recent years is
130 the cellular automaton (see [10], [11], [12] and [13]). The causal model described in this article also
131 represents a spatial causal model.

132 2.1.3. Local causal model

133 The definition of a local causal model presupposes a spatially causal model (see above). A
134 (spatially) causal model is understood to be a local model if changes in the state of the system
135 depend on the local state only and affect the local state only. The local state changes can propagate to
136 neighboring locations. The propagation of the state changes to distant locations; however, they must
137 always be accomplished through a series of state changes to neighboring locations. Special relativity
138 requests that the series of state changes does not occur with a speed that is faster than the speed of
139 light. This requirement is not considered essential for a causal model.

140 Based on a formal model definition of a causal model, a formal definition of locality can be
141 given. A physical theory and a related spatially causal model with position coordinates x and position
142 neighborhood dx (or Δx in the case of discrete space-points) are given. A causal model is called a
143 local causal model if each of the laws L_i applies to no more than a single position x and/or to the
144 neighborhood of this position $x \pm dx$.

145 In the simplest case, this arrangement means that L_i has the form

$$146 L_i : IF c_i(s(x)) THEN s'(x) = f_i(s(x));$$

147 The position reference can be explicit (for example, with the above simple case example) or implicit by
148 reference to a state component that has a well-defined position in space. References to the complete
149 space of a spatially extended object or to a property of a spatially extended object are considered to
150 violate "space-point-locality". Causal models with a system state that includes composite objects with
151 global properties (e.g., mass, charge, velocity) may still be considered as local causal models, more
152 specifically "object-local causal model", even if such global properties are referenced in the model.

153 2.1.4. Background-independence

154 Background independence is an important requirement that is typically established for spacetime
155 models such as spin networks, spin foam, and causal dynamical triangulation. This requirement seems
156 to be mandatory for a local causal spacetime model that supports the emergence of spacetime from a
157 minimal or zero source. Background independence means that all spacetime dynamics, in particular
158 the emergence of space, must be expressible without reference to any predefined coordinate system or
159 other global spacetime properties. For a causal model, this means that the structure of spacetime must
160 not contain components and properties that are non-local.

161 2.1.5. Composite objects

162 Models of areas of physics typically contain spatially extended composite objects such as particles,
163 atoms, stars, and so forth, and typically object-global properties (e.g., mass, charge, velocity) are
164 referenced in such models. According to the definition of a local causal model (above), such models
165 may only be called "object-local causal models" (as opposed to "space-point-local causal models"). Such
166 models may be useful; however, care must be taken that the assignment of object-global properties
167 to composite objects is admissible with the level of accuracy aimed for. Object-global properties are
168 typically the result of aggregations from lower-level relationships. The aggregations toward a single
169 global attribute value may be admissible with classical physics, but questionable with refinements of
170 modern theories of physics. A famous example of the inclusion of global object properties refers to the
171 attributes of mass and charge with quantum field theory when particles are no longer considered to be
172 point-like particles.

173 3. The elementary structure of spacetime

174 3.1. The space-time relationship

175 With GRT and SRT, space and time are said to be integrated into spacetime. For a GRT-compatible
 176 model of spacetime dynamics, the integration of space and time remains visible, but with a different
 177 interpretation. With GRT, the integration of space and time is mathematically expressed in the usage
 178 of tensors (e.g., curvature tensor) and 4-vectors with a time component and spatial components.
 179 Physically, the integration is reflected, among other ways, in the metric and the symmetries that hold
 180 for the combined (space+time) entities and the corresponding laws of physics.

181 In the proposed causal model of spacetime dynamics, the tensors and 4-vectors of GRT/SRT
 182 occur only as the starting point for the introduction of GRT-compatible equivalent model parameters.
 183 The integration of space and time appears to be disturbed by the fundamentally different roles space
 184 and time represent in a causal model. Time and the progression of time are an inherent feature of the
 185 physics engine of the causal model. The physics engine implements the uniform and simultaneous
 186 progression of time. Space is the explicit global object that is part of the system state. Other objects of
 187 the system state are positioned in space. Although space and time conceptually have quite different
 188 roles within the causal model, it is their mutual relationship that establishes their (re-)integration.

189 In GRT, the curvature specification, i.e., the curvature tensor, contains, in addition to the three
 190 space-related components, a time-related component. As an example of the impact of the time factor,
 191 the gravitational redshift is explained as the consequence of the time factor in the spacetime curvature
 192 (see, for example, [14], page 231).

$$\Delta s^2 = -\left(1 - \frac{2GM}{c^2 r}\right)(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 \quad (1)$$

193 This means a clock at position (x, y, z) would run by a factor

$$F_1 = \sqrt{1 - \frac{2GM}{c^2 r}} \quad (2)$$

194 slower than a clock that is not affected by a gravitational field. A standard clock at some point A of
 195 low potential (for example, on the surface of the earth) would go slower than the same clock at point B
 196 of higher potential (for example, at a GPS satellite). In [14]: "... The gravitational redshift implies that
 197 time itself runs slightly faster at the higher altitude than it does on the Earth." For the GPS system, the
 198 difference is 45 microseconds per day: This is the rate at which the clocks at the satellites go faster (see
 199 [15]). In GRT, this effect is called "gravitational time dilation". For reasons that are described in the
 200 following, the author prefers the wording (gravitational) "clock rate dilation".

201 For a mapping of the time factor of the GRT curvature specification to the proposed spacetime
 202 model, two problems arise:

- 203 1. In the causal model, the clock rate (i.e., the proper time) is a property of the whole causal
 204 subsystem. The assignment of clock rates to the different positions occupied by a spatial
 205 distributed causal subsystem is not supported with the proposed causal model. The assignment
 206 of differing clock rates to the different positions occupied by a spatial distributed causal
 207 subsystem would make causal models for the dynamics of subsystems extremely difficult.
- 208 2. In the causal model, the clock rate is maintained by the physics engine (i.e., the clock is part of
 209 the physics engine which delivers the uniform state update time interval). Changes in the clock
 210 rate resulting from the objects motion in space would mean that the clock of the physics engine
 211 has to run slower or faster depending on the object's position in space. This would require a
 212 rather ugly interface between the space and the physics engines of the causal subsystems.

213 Problem (2) may be viewed as a problem due to the specific definition of a causal model given in
 214 Section 2. However, there are (good) reasons for this definition of a causal model. Problem (1) refers to

215 the causal model of causal subsystems in general. It would also be difficult to avoid this problem with
 216 alternative causal model concepts.

217 A possible solution that would make it possible to maintain a uniform progression of the state
 218 update time interval SUTI while enabling non-uniform clock rates may be found if one remembers
 219 that, in SRT and GRT, space and time are considered as an entity and that this implies that space
 220 intervals and time intervals can be jointly transformed by certain symmetry transformations. For
 221 the example gravitational redshift, this means that the redshift is interpreted as the dilation of the
 222 wave length instead of the increase of the frequency and that the length dilation affects not only the
 223 wave length but all lengths within the gravitational potential. For the proposed model of spacetime
 224 dynamics, it is assumed that

225

226 **Proposition 1.** *Lengths within the gravitational field are dilated by the factor F_1 .*

227 ¹ How can this help to prevent the need for the dynamic and position-dependent change of the
 228 state update time interval (SUTI)? A further proposition was introduced:

229 **Proposition 2.** *Physical processes run faster/slower depending on the length scale at the position where the
 230 respective physical process executes.*

231 Notice that the clock rate dilation concerns physical processes, not the spacetime structure.
 232 Space(-time) curvature is the result of length dilations. Clock rate dilation is another consequence of
 233 length dilations.

234 The major process that demonstrates the fixed relationship between the length dilation and the
 235 process change rate is the propagation of light. This (simple) process is used as a measure for the
 236 change rate of other processes by setting the speed of light to be a constant c . The next class of
 237 processes where the change rate depends on the length dilation in precisely the proportions as with
 238 the propagation of light are clocks in differing realizations.

239 In summary, in the model of spacetime dynamics, there is no direct reflection of time dilation as a
 240 spacetime attribute. Clock rate dilation (rather than time dilation) occurs as a property of processes
 241 running within space. The clock rate dilation factor can be derived from the length dilation factor F_1 of
 242 the space points where the respective process is currently executing.

243 In the model of spacetime dynamics, two levels of time are distinguished, which in GRT/SRT are
 244 seen as an entity:

- 245 1. At the basic level, the progression of time is associated with the physics engine of the causal model.
 246 The time of the physics engine proceeds in uniform state update time intervals. Simultaneousness
 247 is assumed for all state changes occurring at the same state update cycle.
- 248 2. Differing clock rates, proper times, and relativity of simultaneousness are not associated with the
 249 basic overall spacetime, level (1), but are associated with objects residing and moving in space -
 250 more precisely, with processes running in these subsystems.

251 With space, two levels also may be distinguished, but these are two levels of consideration:

- 252 • At the abstract level (i.e., mathematical level), the space consists of a set of interconnected space
 253 points (see Section 3). Whether or not the totality of interconnected space points represents an
 254 Euclidean space or a specific topology (e.g., Riemann manifold) is left open.

¹ "Gravitational length dilation" appears to be a very controversial subject among physicists (see various discussion in internet forums). The author here takes a strong position while at the same time stating a clear relation between (1) the length dilation and (2) the clock rate dilation, namely by saying that (2) is a consequence of (1).

- 255 • At the physical level (i.e., the essential level), meaning is assigned to the components of the space
256 point. Especially, the length of the connections is no longer a geometrical property, but specifies
257 the Δlength *only* with respect to a specific physical process executing at the respective space point
258 for the time interval SUTI. The process that is used as the measure for the specification of the
259 length is the propagation of light.

260 Thus, the integration of space and time into spacetime is established in the model of spacetime
261 dynamics by the physical meaning assigned to the components of the space points and their
262 connections.

- 263 1. Time progresses uniformly in constant units. As a suitable basic unit of time progression, the
264 state update time interval (SUTI) of the physics engine is taken. This means, the SUTI is assumed
265 to be a system constant.
- 266 2. Length specification is expressed in relation to the spatial distance change caused by a specific
267 physical process running for the duration of the standard unit of time (i.e., the SUTI). This means,
268 in the causal model, spatial distances are not primarily a geometrical property, but rather a
269 physical property used to formulate interrelationships between objects in space.
- 270 3. The physical process that is used as the measure for the standard unit of time as well as the
271 measure of spatial distances is the propagation of light. This has the consequence, that in the
272 model (as with most models of physics), the speed of light c is a constant.

273 The proposition (fact?) that there is such a simple relationship between the spatial length dilations
274 and the rate of state changes of processes that execute at a given position in space is the root of the
275 space-time integration in the proposed model of spacetime dynamics. A possible foundation of this
276 supposed space-time relationship (reflecting the space-time integration) may be that

277 **Conjecture 3.1.** All physical processes can ultimately be broken down to length-related state changes,
278 and changes in the length scaling therefore directly result in clock rate dilations of the affected process.

279 3.2. *The elementary structure of space*

280 The proposed elementary structure of spacetime constitutes the base for the overall model of
281 spacetime dynamics that is compatible with GRT. A number of works toward the same or a similar
282 goal have been published. The work that shows the most similarities with the model described in this
283 article in terms of the overall orientation (background independence; discreteness of time, space, and
284 paths; expressing causal relationships) is causal dynamical triangulation (CDT, see [4], [5], and [6]).
285 The spacetime structure of the model described in this article is based on CDT. However, it was felt
286 that adaptations were required to further refine the causal relationships of spacetime dynamics, in
287 particular to construct a causal model of the emergence of space from a single source.

288 With CDT, the basic space elements are n -dimensional simplexes (e.g., triangles, tetrahedrons; see
289 Fig. 1). In contrast to CDT, the proposed causal model of curved discrete spacetime considers only
290 3-dimensional space elements, i.e., tetrahedrons. The time dimension is treated separately within the
291 causal model. In addition, the elementary units that represent the total space are not (as with CDT) the
292 n -dimensional simplexes, but only the space points together with their connections to neighboring
293 space points. The reason for this simplification was that it was not possible to build up a larger space
294 object by the continuous addition of uniform regular tetrahedrons and (2) the uniformness of the
295 tetrahedrons is obsolete with the proposed model (see Section 4). Whether the space points together
296 with the connections establish specific 2-dimensional surface areas (e.g., triangles) and 3-dimensional
297 solids (e.g., tetrahedrons) is initially left open.

298 **Definition 1.** $\text{Space} := \{ \text{spacepoint } \dots \};$
299 $\text{spacepoint} := \{ \psi, \text{dilation factor}, \text{connections} \};$
300 $\text{connections} := \{ \text{connection}_1, \dots, \text{connection}_n \};$
301 $\text{connection} := \{ \text{neighborspacepoint}, \text{direction}, \Delta\text{curvature} \};$

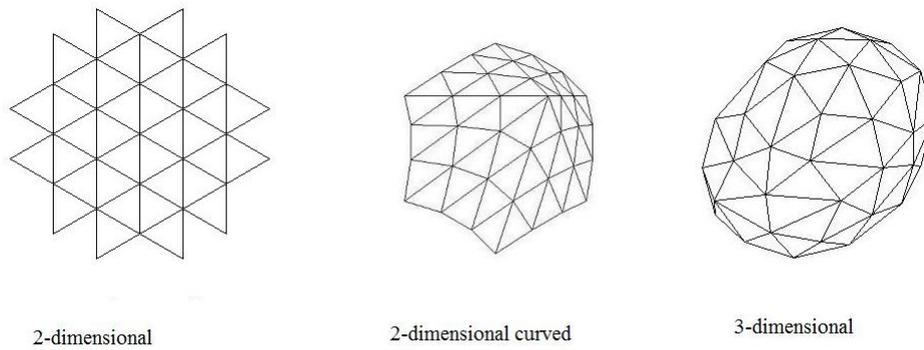


Figure 1. Elements of spacetime of Causal Dynamical Triangulation.

302 ψ is the physical content that is directly associated with the space. These are the fields residing
 303 in space. As with spin networks, spin foam networks, and causal dynamical triangulation, each
 304 space point is connected with a number of other space points via "connections" (i.e., edges in CDT). A
 305 connection carries the information about the connected neighbor space point, the connection direction,
 306 and the propagation gradient of the curvature changes (see Section 4).

307 All the information associated with the space point is local to the space point (i.e., no globally
 308 defined position or direction specification). This supports the background independence of the
 309 spacetime model.

310 To enable the determination of the spatial distance between two space points, some information
 311 about the distance between neighbor space points is required. This could be provided, for example,
 312 in form of position coordinates (Provision of space point coordinates would violate background
 313 independence). or by the specification of the lengths of connections between the neighbor space points.
 314 In support of a causal model of the movement of objects in curved space, for the proposed model of
 315 spacetime dynamics, it is defined that

316 **Proposition 3.** *The length of the connections between space points is a constant;*

317
$$L_{\text{connection}} = c \cdot \text{SUTI}.$$

318 ² The overall distance between two space points within the curved space is then obtained by
 319 multiplying $L_{\text{connection}}$ by the number of space points k_p on the geodesic path from space point-1 to
 320 space point-2. Length dilation within a gravitational potential as assumed by Proposition 1 in Section
 321 3.1, is realized by the appropriate arrangement of the space points within space (see Section 4).

322 Proposition 3 is, first of all, a physical statement, although it has consequences for the space
 323 geometry. The physical statement is:

324 *The (spatial) distance that light moves during a state update time interval (SUTI) is equal to the distance*
 325 *between two connected neighbor space points, which is equal to the distance by which space expands during a*
 326 *SUTI.*

327 The geometry of the emerged space (e.g., whether an Euclidean space or a Schwarzschild metric
 328 emerges) depends on the space expansion algorithm. With the proposed model of spacetime dynamics
 329 the resulting geometry depends on the ratio by which the number of space points grow at a single
 330 expansion step (see Section 4.1).

² In combination with the other features of the proposed spacetime model, Proposition 3 results in a certain spacetime curvature. The complete GRT-compatible spacetime curvature will be introduced in Section 3.3.

331 3.3. The representation of space(-time) curvature

332 Space curvature is a major ingredient of GRT. In GRT, specifically in Einstein's equation

$$333 G^{\alpha\beta} = \frac{8\pi G}{c^4} T^{\alpha\beta},$$

334 space curvature is expressed by the curvature tensor $G^{\alpha\beta}$. Thus, the simplest solution would be to say
 335 that a space-curvature component is assigned to the space point and that this curvature specification
 336 provides the same information as the curvature tensor of GRT. However, some adaptations appear
 337 reasonable. In Section 3.2 above, the space component of the system state is specified as consisting
 338 of a set of space points, and, at the next level of detail, a space point is specified as consisting of
 339 dilationfactor, connections, and the space content ψ .

340 $\text{spacepoint} := \{ \psi, \text{dilationfactor}, \text{connections} \};$

341 The dilationfactor supports the generation of the space curvature with the propagation of space
 342 changes (including the emergence of space). Once the space has emerged, the space(-time) curvature
 343 is represented by (1) the distribution and density of the space points and (2) the (spatial) distances
 344 between neighboring space points. Proposition 3 (above) states that the length of the connections
 345 between space points, i.e., the distances between neighboring space points, is a constant. Thus, the
 346 main parameter that determines the space curvature is the density distribution of the space points.
 347 The density distribution of space points is realized by the appropriate arrangement of the space points
 348 within space.

349 As described in Section 3.1, Proposition 2, the the clock rate dilation (i.e., the time-related
 350 component of the GRT curvature) is a consequence of the length dilations. This means that the
 351 information which specifies the length dilations implies the time-related component of the GRT
 352 curvature.

353 4. Space(-time) dynamics

354 The dynamics of spacetime is triggered by the minimal sources, called "quantum objects". With
 355 each update cycle of the system state a new space change action starts at each quantum object. The
 356 space changes propagate from the quantum objects through the whole space in steps according to the
 357 update cycles of the physics engine. In support of a *local* causal model, with each update cycle, the
 358 space changes propagate only to (part of) the neighboring space points. The propagating space changes
 359 always have definite directions at each space point, from the "in-connections" to the "out-connections"
 360 of the space point. The out-connections of space point sp , at a given update cycle i , are in-connections
 361 of some neighbor space points of sp with the subsequent update cycle $i+1$.

362 The directions of space changes, i.e., the identification of in/out-connections, are determined
 363 by the $\Delta\text{curvature}$ attribute of the space point connections. For a given space point, only part of
 364 the connections can be in-connections, which means $\text{connection}.\Delta\text{curvature} > 0$. The remaining
 365 connections of the space point are out-connections.

366 The overall process of space change propagation is specified as

367 **Specification 1.** $\text{spaceprogression}() := \{$
 368 FOR (all space points sp_i) {
 369 IF (inconnections(sp_i) {
 370 propagateOUT(sp_i);
 371 } }
 372 }

373 4.1. The emergence of space from a single source

374 The space that emerges from a single source represents a Schwarzschild metric. In the causal
 375 model, the large-scale space object emerges by the successive addition of surface layers to the initial
 376 space object.

```

377
378 SSspaceemergence( source ) ::= {
379     spaceobject ← source;
380     DO UNTIL(nonContinueState(S)){
381     spaceobject ← extendbynexlayer(spaceobject);
382     }
383 }

```

384 For the refinement of the above space emergence process, answers to the following questions have to
 385 be provided:

- 386 1. What are the elementary units of space?
- 387 2. How does the initial space object look like?
- 388 3. What is the detailed algorithm for *extendbynexlayer(spaceobject)*?

389 4.1.1. The elementary units of space

390 The elementary structure of space, including the elementary units of space, have already been
 391 described in Section 3.2. In the proposed model, the elementary units of space are the space points
 392 together with their connections to neighbor space points (see Definition 1). The number of connections
 393 (and thus the number of neighbor space points) of a given space point must be large enough to span
 394 the complete three-dimensional space. It should be small enough to enable a moderate growth of the
 395 number of space points with the chosen algorithm of the space emergence process. In the model, a
 396 typical space point has 14 connections (see Fig. 2):

- 397 • source connection: one connection towards the source of the emerging space,
- 398 • target connection: one connection in the primary emerging direction,
- 399 • surface connections: four connections in the plane that is perpendicular to the source connection
 400 (S1, S2, S3, S4 in Fig.2),
- 401 • four connections in between the source connection and the surface connections (A1, A2, A3, A4
 402 in Fig.2),
- 403 • four connections in between the target connection and the surface connections (B1, B2, B3, B4 in
 404 Fig.2).

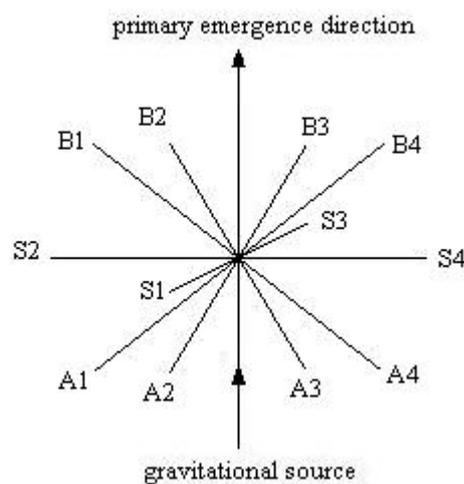


Figure 2. The 14 standard connections of a space point.

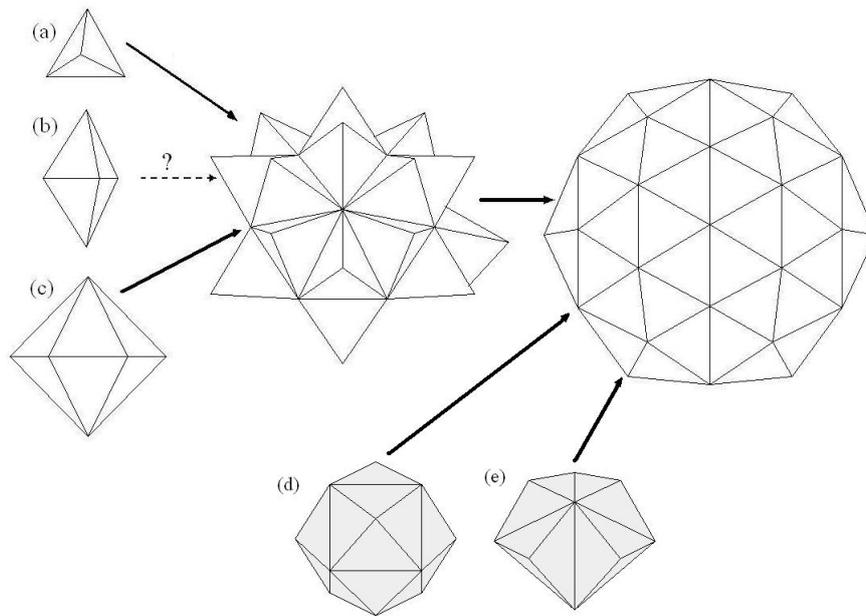


Figure 3. Alternative initial space elements.

4.1.2. The initial space object

There are several alternatives for the initial space object from where the emergence of space and the propagation of gravitational space dynamics may start. Fig. 3 shows a number of alternatives investigated by the author. The simplest solution would be to have the space emergence process, starting from a single tetrahedron (case (a) in Fig. 3) or a double-tetrahedron (case (b) in Fig. 3). However, more symmetrical initial space objects, such as case (c) or case (d) enable the early emergence of a symmetrical larger space object through simple space extension algorithms. For the present model of spacetime dynamics the initial space object is a single space point surrounded by 14 neighbor space points and the respective connections. The 14 neighbor space points, together with the interconnections among them represent a spherical surface - the initial surface from where the space emergence starts (case (d) in Fig. 3).

4.1.3. The space expansion algorithm- *extendbynextlayer(spaceobject)*

As described above, space emergence from a single source is a continuous process where each system state update cycle of the causal model adds another layer of space to the existing space object. This means, with each expansion step st_i a number kp_i of new space points is generated. The new space points are interconnected with their respective neighbor space point, forming kt_i surface triangles. Various kinds of space expansion algorithms are possible. The key differentiating parameters for the alternative space expansion algorithms are the growth factor gp of the number of surface space points (i.e., $kp_i = gp \cdot kp_{i-1}$) and the related growth factor gt of the number of surface triangles (i.e., $kt_i = gt \cdot kt_{i-1}$). Table 1 shows the major parameters for an example space emergence algorithm that starts with an initial space object with 12 surface triangles (case (c) in Fig. 3). The surface growth factor $gt = 3$, i.e., $kt_i = 3 \cdot kt_{i-1}$. The number of surface space points increases by the number of surface triangles, $kp_i = kp_{i-1} + kt_{i-1}$.

Further parameters shown in Table 1 are the total number of space points, the radius r_i of the surface and the average edge length, L of the surface triangles. The average edge length, L is the length measured by the author's computer simulations and these computer simulations and the length measurements assume *Euclidean space*. However, the space emergence process of the model of spacetime dynamics has to generate curved space that adheres to Schwarzschild metric, with length dilations in

Table 1. Layers of space expansion, constant surface $\Delta r = 1.0$

Layer number	surface triangles, kt	surface points, kp	total points, kpt	radius, r_i	av. edge length, L
0	12	8	8	1.00	1.63
1	36	20	72	2.00	1.72
2	108	56	228	3.00	1.55
3	324	164	660	4.00	1.22
4	972	488	1956	5.00	.88
...
12	6377292	3188648	12754596	13.00	...
13	19131876	9565940	38263764	14.00	...
14	57395628	28697816	114791268	15.00	...
...
i	$3 \cdot kt_{i-1}$	$kp_{i-1} + kt_{i-1}$	$ks_i + 3kp_{i-1}$	$(i + 1)100$	

4.33 accordance with the Propositions 1, 2 and 3. Especially, Proposition 3 says that $L_{connection}$ is constant.
 4.34 With the example shown in Table 1, $L_{connection} = \Delta r = 1.0$. This means that the circumference of a
 4.35 surface, if curved space and $L_{connection} = 1.0$ is assumed, depends solely on the number of surface
 4.36 space points, kp_i . The number of surface space points, kp_i for a surface S_i is determined by the space
 4.37 expansion algorithm. For the proposed model of spacetime dynamics, a curved space with length
 4.38 dilations according to F_1 at the surfaces (see Eq. 2) has to emerge. This can only be achieved with
 4.39 a decreasing growth factor gp . The space expansion algorithms that have been investigated by the
 4.40 author showed that with the proposed model, GRT compatible space expansion algorithms are feasible.
 4.41 However, unless the algorithm gets unnaturally complex, occasional inhomogeneities seem to be
 4.42 unavoidable. In particular at the very small scale, i.e., near the minimal gravitational sources, it
 4.43 appears to be difficult or impossible to preserve the GRT compatible behaviour. The surrender of
 4.44 perfect GRT compatibility at the very small scale may avoid singularities that occur with the differential
 4.45 equations of GRT.

4.46 4.2. The propagation of space changes caused by multiple sources

4.47 The assumption that space changes start at the minimal sources implies that the aggregation of
 4.48 space changes from many sources is the normal case. The model of the propagation of space changes
 4.49 that are caused by multiple sources is based on the single-source propagation (Section 4.1). The
 4.50 aggregation of the single-source propagations has to be accomplished by a local causal process, i.e.,
 4.51 by a series of aggregations of neighboring space changes. Only long range, this dynamical process,
 4.52 can achieve overall gravitational space changes (i.e., curvature changes) that are compatible with the
 4.53 predictions of GRT and Newtonian dynamics.

4.54 To simplify the description, in this article, "multiple sources" is initially equated to "two sources".
 4.55 In simple cases, the treatment of many sources can be performed by a series of two source propagation
 4.56 processes.

4.57 For the overall two-source propagation process, three phases can be distinguished:

- 4.58 • Phase-1, the phase where the changes from the two sources propagate independently.
- 4.59 • Phase-2, the phase where the changes start to overlap and therefore have to be aggregated.
- 4.60 • Phase-3, the phase where the aggregated changes propagate like single source changes.

4.61 Fig. 4 shows an example snapshot in two dimensions, with the areas that are covered by phase-1 and
 4.62 phase-3 roughly indicated. Notice that the 2-dimensional representation in Fig. 4 is a simplification
 4.63 which is misleading with certain more detailed considerations.

4.64 A major assumption of the proposed model is that the propagation that occurs at a space point
 4.65 sp has a definite (consolidated) in-direction and the same (overall) out-direction. The consolidated

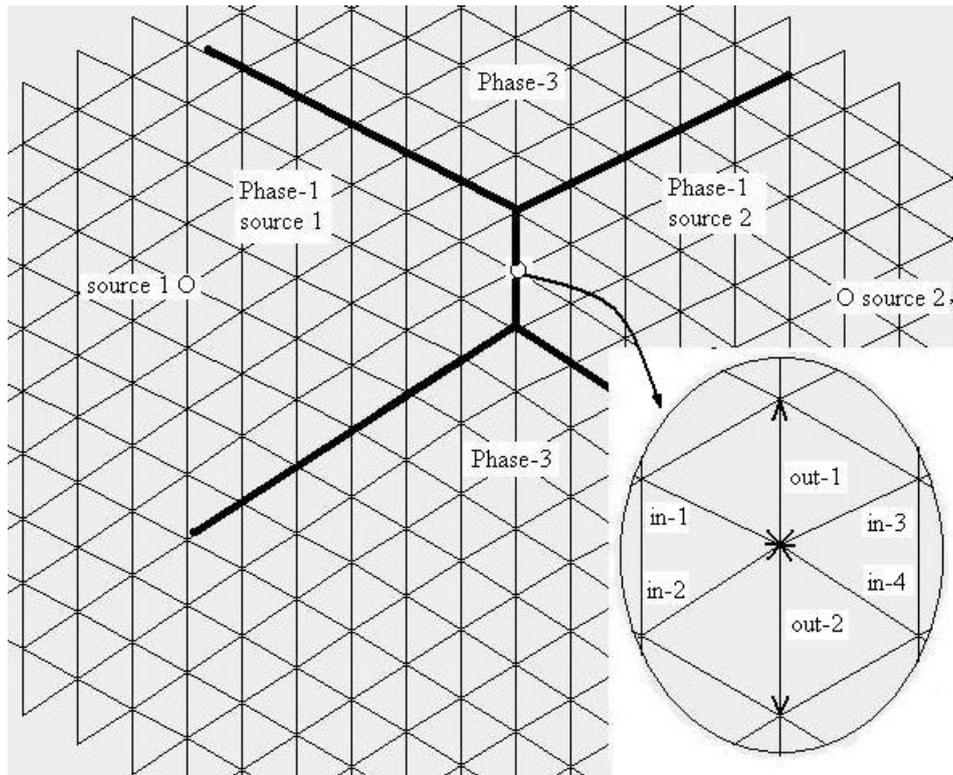


Figure 4. Propagation of space changes caused by 2 sources.

466 in-direction is the vector sum of the multiple in-connections. The overall out-direction is distributed
 467 over the multiple out-connections.

468 4.2.1. Phase-1:

469 The propagation of space changes prior to the points where the changes meet is exactly the single
 470 source propagation described in Section 4.1.

471 4.2.2. Phase-2:

472 When the space changes originating from (two) different sources meet at space point sp , the
 473 changes that arrive from n space point connections ($n \geq 2$) are summarized into a single out-vector.
 474 The out-vector is then distributed to the out-connections (see Fig. 4, the magnifying glass area).
 475 If there are no out-connections left – i.e., if all connections of sp are in-connections – the weakest
 476 in-connection(s) are taken as out-connection(s).

477 4.2.3. Phase-3:

478 After the changes from the multiple sources are summed up, the further common propagation
 479 of the space changes continues like the single-source propagation (Section 4.1). As a special case,
 480 the phase-3 propagation may collide with phase-1 propagation from one of the two sources. With
 481 the proposed model of spacetime dynamics, the collision of space changes is handled like a phase-2
 482 propagation, described above.

483 Compatibility with classical, i.e., Newtonian dynamics evolves during phase-3. The compatibility
 484 with classical dynamics is reflected in mainly the following items:

- 485 1. It is valid to assume an aggregated mass M_{aggr} that represents the aggregation of the masses of
 486 the sources of the space changes.

- 4.87 2. It is valid and possible to identify a position in space where M_{aggr} is assumed to be located. The
 4.88 position is usually called the "center of mass".
 4.89 3. The (single) aggregated mass M_{aggr} is the sum of the masses of the sources of the space changes.

$$4.90 \quad M_{aggr} = M_1 + \dots + M_n.$$

4.91 Only when the propagation of space changes reaches a certain distance r from the center of mass
 4.92 that the aggregated mass $M_{aggr}(r)$ can be equated to the sum of the masses of the sources.

4.2.4. Aggregation of space dynamics from $n \gg 2$ sources

4.94 The above-described model of the space dynamics aggregation from two sources, with the three
 4.95 aggregation phases shows that compatibility with classical dynamics will only evolve at the end of
 4.96 phase-3. Prior to that stage, inhomogeneities, i.e., areas where only a subset of the gravitational source
 4.97 participates in the aggregation, will occur (and will not disappear during the continued propagation of
 4.98 space changes). If the aggregation of space dynamics applies to $n \gg 2$ sources, further inhomogeneities
 4.99 may exist, depending on the distribution of the sources within the space. If the distribution of the
 5.00 sources establishes gravitational sub-clusters such as solid bodies, planets or stars, where it is possible
 5.01 to assign an aggregated mass M_{aggr} and a center of mass, the sub-clusters may represent a gravitational
 5.02 source at the next higher level.

5.03 5. Applications of the model of spacetime dynamics to quantum field theory

5.04 In Sections 3 and 4, a causal model of the dynamics of spacetime has been described. According
 5.05 to the model, spacetime changes (i.e. the gravitational field) continuously propagate from the minimal
 5.06 sources, called quantum objects. In quantum field theory (QFT), the quantum objects are also the
 5.07 sources of additional dynamical processes. Quantum objects are the sources of virtual particle
 5.08 fluctuations. The movement of quantum objects through space, is described in terms of paths that
 5.09 constitute the wave function. Also, particle scattering in QFT is described in terms of paths for
 5.10 virtual particles that lead to a range of probability amplitudes for different possible scattering results.
 5.11 Considering the various cases of the dynamics in QFT, the question arises on how QFT (virtual particle)
 5.12 paths relate to the model of spacetime dynamics described in the preceding sections. As with the
 5.13 general subject, the question can be asked in two parts:

- 5.14 1. How do the dynamics of quantum fields (including quantum objects) relate to the elementary
 5.15 structure of spacetime described in Section 3?
 5.16 2. How do the dynamics of quantum fields (including quantum objects) relate to the model of the
 5.17 dynamics of spacetime described in Section 4?

5.18 Question 1 requests a detailed answer in order to demonstrate that the model of spacetime dynamics
 5.19 can also be applied to the dynamics of quantum fields and quantum objects. The details are straight
 5.20 forward, yet non-trivial. The answer to question 2 is less direct. Integrating quantum field dynamics
 5.21 and spacetime dynamics at different degrees are imaginable, ranging from minimal integration (i.e.,
 5.22 adaptation to the proposed spacetime structure only) to maximal integration (i.e. a combined model
 5.23 for both subjects as for example quantum gravity aims for). In Section 5.2 the model proposed by the
 5.24 author is described.

5.25 5.1. Mapping quantum fields and quantum objects to the elementary structure of spacetime

5.26 The task is to map the parameters and components that constitute a quantum field or a quantum
 5.27 object to the parameters and components of the spacetime as described in Section 3. In Section 3.1,
 5.28 the integrated view of spacetime (as assumed in standard GRT) is described as being disturbed by
 5.29 the strict separation of space and time implied by the *causal* model. This may be considered as too
 5.30 restrictive for a GRT-compatible model of spacetime to be applied to QFT. However, starting with a
 5.31 topology where space and time are separated into $\Sigma \times R$ where Σ is a three-dimensional manifold and
 5.32 R is a line, is a popular approach with theories directed toward quantum gravity (see [16] on loop

533 quantum gravity). It leads to the so-called "Hamiltonian formulation of general relativity" (see [16]).
 534 As with the model of spacetime dynamics described in this article, the integrated view of spacetime is
 535 restored by processes that relate the spatial changes to the progression of time (e.g., by a causal model).

536 In Section 3.2, Definition 1, space is defined as consisting of interconnected space points and a
 537 space point is defined as

538 $\text{spacepoint} := \{ \psi, \text{dilationfactor}, \text{connections} \}.$

539 Here, fields are represented by the component ψ . (Whether ψ refers to a single type of field or to
 540 possibly multiple field types is here left open.) In [3], quantum objects are defined as composite objects
 541 consisting of 1 to n particles.

542 **Definition 2.** $\text{quantumobject} := \{$
 543 $\text{globalquantumobjectattributes};$
 544 $\text{particle}_1,$
 545 \dots
 546 $\text{particle}_n;$
 547 $\}$

548 The particle encompasses a set of spacepoints and $\text{globalparticleattributes}$:

549 **Definition 3.** $\text{particle} := \{$
 550 $\text{globalparticleattributes};$
 551 $\text{spacepoint}_1,$
 552 \dots
 553 $\text{spacepoint}_k;$
 554 $\}$

555 Examples of global attributes ($\text{globalquantumobjectattributes}$ and $\text{globalparticleattributes}$) are
 556 mass, charge, spin, etc. With the specification of a *local* causal model at a specific level of detail, the
 557 inclusion of the global attributes may disturb the provision of a *local* causal model. Therefore, in a
 558 detailed local causal model, the global attributes may have to be supported by aggregation processes
 559 and/or collective behaviour processes (see Section 5.3 Collective behaviour).

560 5.2. Mapping of the dynamics of quantum fields and quantum objects to the dynamics of spacetime

561 The model that is roughly described as follows is based on two types of work:

- 562 1. Loop quantum gravity [16] and its descendants comprising spin networks (see [17]), spin foam
 563 (see [18]), and causal dynamical triangulation [4].

564 The coupling of the dynamics of space (e.g., the propagation of space changes) with the dynamics
 565 of quantum fields and particles is an idea that has already been pursued with causal fermion
 566 systems (see [20]).

- 567 2. In [1] and [3] a causal model of QT/QFT is proposed where the physics of QT/QFT is confined
 568 in "quantum objects". The refinement and an improved foundation of the model described in [1]
 569 and [3] was determined to require a causal model of spacetime dynamics. The causal model of
 570 spacetime dynamics described in Sections 3 and 4 has been developed as an attempt to fulfill this
 571 requirement.

572 5.2.1. The movement of objects within space

573 According to GRT, the movement of objects within space follows the geodesics of the space.
 574 This means, two parameters determine the path of the object: (1) the objects momentum and (2) the
 575 structure of the space, in particular, the curvature of the space.

576 With the model of spacetime dynamics, especially when applied to quantum theory, the
 577 GRT-based model of object movement has to be adjusted and refined for two aspects: (1) the term
 578 geodesics must be redefined for discrete granular paths, and (2) the momentum of quantum objects in
 579 general does not have a single definite value, but a range of (possible) values. Regarding these two
 580 aspects resulted in the following model for the movement of objects within space.

- 581 • The moving object is represented by the space content ψ of a set of space points (see Definition 1).
- 582 • Part of ψ is the momentum vector component p .
- 583 • When the propagation process reaches a space point sp , the momentum vectors from the
 584 in-connections of sp are summarized to a single consolidated momentum vector.
- 585 • The consolidated momentum vector is then distributed to the out-connections.
- 586 • The distribution is such that the out-connection(s), which matches best the direction of the
 587 consolidated momentum vector, obtains the largest part of the consolidated vector.

588 Given the aforementioned schema, the following types of object movements may be distinguished:

- 589 1. Classical straight forward movement following a single definite geodesic,
- 590 2. Quantum movement with a network of paths and with different probability amplitudes,
- 591 3. Loops
 - 592 (a) Classical loops according to a geodesic that represents a loop (examples: planets and
 593 satellites),
 - 594 (b) Quantum loops: Loops resulting from quantum objects and constituting quantum objects.

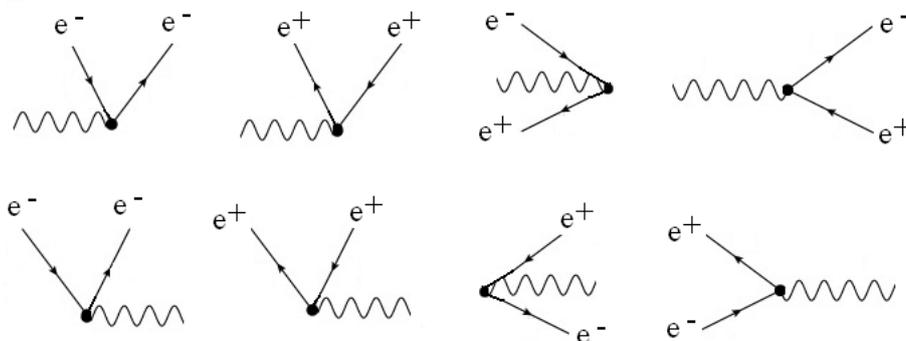
595 Quantum movements and quantum loops are further described in the following.

596 5.2.2. Quantum movement

597 In Section 4, the dynamics of spacetime is described as involving the summation of the
 598 in-connections of a space point followed by the distribution of the aggregated effect to the multiple
 599 out-connections. A similar operation is also known with the operator equations of QFT (see, for
 600 example [21]). Two virtual particles may join and annihilate each other to create a single new virtual
 601 particle of a specific type; or vice versa, a single virtual particle may be annihilated resulting in
 602 the creation of two new virtual particles of specific types. The graphical representation of the
 603 possible annihilate/create (or join/split) operations is given by Feynman diagrams. In quantum
 604 electrodynamics (QED), the operator equation for the creation and annihilation of the field has the
 605 form (see [21]):

$$606 H_W(x) = -eN\{(\bar{\psi}^+ + \bar{\psi}^-)(\not{A}^+ + \not{A}^-)(\psi^+ - \psi^-)\}_x$$

607 where $\psi^+, \psi^-, \bar{\psi}^+, \bar{\psi}^-, \not{A}^+, \not{A}^-$ are the creation and annihilation operators for electron, positron
 and photon. This leads to the eight fundamental Feynman diagrams shown in Fig. 5. The operator



608 **Figure 5.** Fundamental Feynman diagrams of quantum electrodynamics.

609 combination of QFT (normally) applies to three operations (two creates and one annihilate or one
 610 create and two annihilate). For a mapping of the QFT processes to the model of spacetime dynamics,
 the QFT operations have to be mapped to the n in/out connections of the space point. A typical

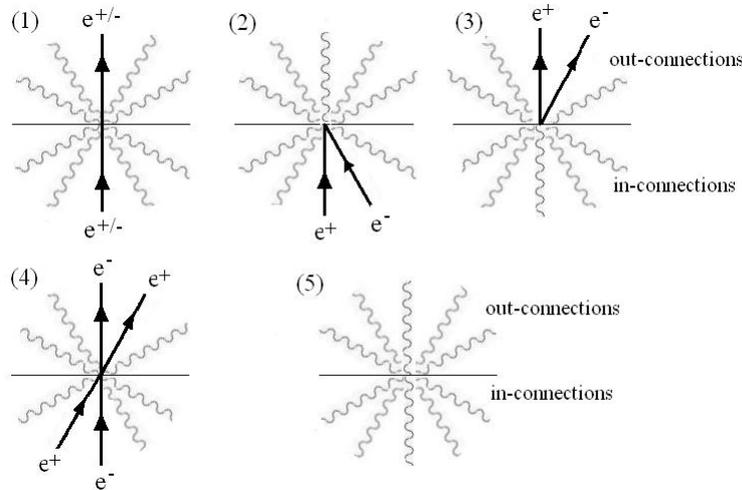


Figure 6. Possible QED connections of a space point.

611 space point has $n = 14$ connections. This enables the use of various strategies (i.e. algorithms) for
 612 the mapping of the three lines of a fundamental Feynman diagram to the 14 space point connections.
 613 For the application of the model of spacetime dynamics to quantum fields, the overall strategy is the
 614 preservation of the number of fermion in-connections and fermion-out connections and the allowance
 615 of additional boson connections. This enables the types of QED space point connections shown in
 616 Fig. 6. (For practical purposes only part of the boson connections are shown in Fig. 6). The cases that
 617 correspond to the QED first order diagrams shown in Fig. 5 are the cases (1) to (3) in Fig. 6. Case (4)
 618 and case (5) support an increased diversity of the possible fermion and boson paths.

619 Notice that the mapping of the QFT operations to the in/out connections of the space points is
 620 part of the dynamical QFT processes (it is not a static mapping).

621 The utilization of the complete set of in/out connections for the join/split operation on (virtual)
 622 particle paths delivers the equivalent to the superposition of paths which in QFT is expressed by the
 623 path integral. In standard QFT (see [19]), the path integral is written as

$$624 \quad K(b, a) = \int_a^b e^{(i/\hbar)S[b,a]} Dx(t).$$

625 The discreteness of the model parameters (space, time and paths) may results in significant
 626 incompatibilities at the very small scale. The discreteness of the model parameters in conjunction
 627 with the *local causal* model eliminates the need for renormalization (if a suitable algorithm for the
 628 assignment of in/out connections is applied).
 629

630 5.2.3. Quantum loops

631 In terms of a causal model, a physical object moves into a loop, if two conditions are satisfied:

- 632 1. The object moves in a spatial environment that enables *geodesic loops*.
- 633 2. The object has reached a *recurrence state*, i.e., a state such that the causal progression of the object
 634 may lead to a recurrence of this object state.

635 Geodesic loops can occur only if space has a specific curvature. The simplest example of space
 636 curvature that enables geodesic loops are the spherical surfaces that develop with the emergence
 637 of space caused by one or multiple sources (see Sections 3 and 4 and Fig. 7). With the model of
 638 spacetime dynamics, the emergence of spatial changes occurs through the successive addition of

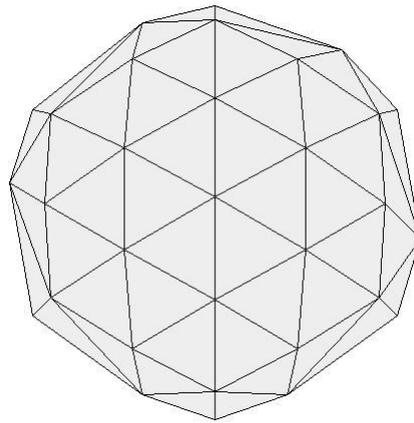


Figure 7. Surface of space that enables geodesic loops.

639 spherical surfaces. The spherical surfaces occur already around the minimal sources, i.e., the quantum
 640 objects. As described above (see Quantum movement), in contrast to GRT, where the geodesics are
 641 single lines, in the model proposed, the geodesic consists of a network of paths with splits and joins
 642 at each space point according to the rules and diagrams of QFT. This holds true also for the geodesic
 643 loops (see Fig. 8). Because of the large number n of in/out connection ($n > 3$, $n \sim 14$), there may be
 paths of the network that do not end up in the loop. In general, there will be open ends (see Fig. 8).

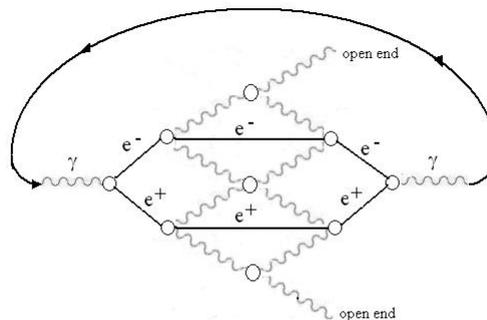


Figure 8. A quantum loop containing a network of paths.

644 In the quantum loop shown in Fig. 8, the in/out connections are labeled by specific symbols.
 645 In Fig. 8 the labels (and thus the paths) refer to (virtual) particle types of QED (γ , e^- , e^+). This
 646 emphasizes the close relationship between the quantum loop network and Feynman diagrams. An
 647 alternative labeling of the paths, and thus an alternative interpretation of the quantum loop, would be
 648 to show the similarity with spin networks. With spin networks, the connections (i.e., line segments)
 649 within the network are attributed by spin numbers (with the original introduction by R.Penrose [17])
 650 or the dimension of the parallel transport matrix (see [16]). Similar to spin networks, the quantum
 651 loop network defines the possible paths of state transitions including possible final result state (i.e., the
 652 recurrence state). If an extra (logical) dimension is added to the quantum loop network (or to the spin
 653 network) to show the complete multitude of possible networks that support a specific recurrence state,
 654 the equivalent to the spin foam (see [18]) is given.
 655

656 5.3. Collective behaviour

657 One of the objectives of the causal model presented in this article is that the model should be a
 658 *local* causal model. The target space-point-locality is damaged by the inclusion of composite quantum
 659 objects with object-global attributes (e.g. mass and spin) and instantaneous processes (e.g., the collapse
 660 of the wave function and entanglement), if it is not possible to break down the formation of the
 661 composite objects and the related non-local effects to space-point-local state transitions. In the causal
 662 model of QT/QFT described in [2], the non-local effects are explained by the collective behaviour of
 663 spacetime elements. Based on the causal model of spacetime dynamics described in Sections 3 and 4
 664 and the concept of the quantum loops, the model described in [2] can now be refined as follows.

665 The formation of (semi-) stable quantum objects (elementary as well as composite quantum
 666 objects) is a collective behaviour process in form of a quantum loop that runs within the (small) area of
 667 curved space around the components of the quantum object.

668 As the described collective behaviour process represents a model for the emergence of quantum
 669 objects and the related quantum-object-global attributes, the disturbance of this collective behaviour
 670 process provides a possible model for the instantaneous non-local QT/QFT processes such as particle
 671 decay, the collapse of the wave function, and decoherence. The model which describes the emergence
 672 of a quantum object as a collective behaviour process has many similarities with G. Groessing's
 673 proposal to explain the emergence of a quantum system as a self-organization process (see [22]).

674 6. Applications of the model of spacetime dynamics to cosmological dynamics

675 In addition to enabling alternative interpretations and models of QFT in curved spacetime
 676 (the topic of Section 5), the proposed model of spacetime dynamics also leads to (possible) new
 677 interpretations and models of cosmological dynamics. The main features of the model of spacetime
 678 dynamics that enable/demand new interpretations are (1) gravitational length dilations and (2) the
 679 non-smooth aggregation of spacetime dynamics.

680 6.1. Gravitational length dilations

681 Propositions 1 and 2 in Section 3.1 state that wherever GRT predicts a time dilation, this time
 682 dilation is accompanied by a length dilation (in fact, the propositions say that the length dilation is
 683 the primary effect, and that the clock rate dilation is a consequence of the length dilation). Applied to
 684 cosmological dynamics, this means that the lengths of the orbits of cosmological objects (e.g. stars,
 685 planets, moons) orbiting a gravitational source are dilated by a factor of $F_1 = \sqrt{1 - \frac{2GM}{c^2 r}}$ (see Eq. 2).
 686 Since the radius r is only dilated by a factor $F_r < F_1$, this means that, contrary to Euclidean geometry,
 687 the circumference C of an orbit around a gravitational source is greater than $2\pi r$. The dilation of the
 688 circumference is an effect which cannot be directly observed by an observer such as an astronomer.
 689 In the projection of the orbit to a picture in Euclidean geometry, the relation between the observed
 690 radius r_o and the observed circumference C_o is still $C_o = 2\pi r_o$. The orbital length dilation can be
 691 observed only indirectly, by examining the dynamics of objects orbiting around a gravitational source.
 692 For example, the velocity of objects orbiting a gravitational source will show deviations from the
 693 laws of Newtonian dynamics. The most famous examples of unexpected deviations from Newtonian
 694 dynamics in cosmological observations are the "flat galaxy rotational curves", for which gravitational
 695 length dilations offer a possible explanation (see below).

696 In Section 3.5, Proposition 2 states that in the proposed model of spacetime dynamics, clock rate
 697 dilation is considered a secondary effect caused by the primary effect, the length dilation due to space
 698 curvature, i.e., due to the gravitational field. According to Section 3.5, the lengths (and, as a secondary
 699 effect, also the clock rates) around a gravitational source are dilated by the factor

$$F_1 = \sqrt{1 - \frac{2GM}{c^2 r}}.$$

This means, that if at a spacepoint near the gravitational source, at radius r_1 , the dilation factor is
 $F_1 = \sqrt{1 - \frac{2GM}{c^2 r_1}}$, at a spacepoint that is farther away from the source, at radius r_2 ($r_2 > r_1$), the dilation

factor gets closer to its maximal value, 1. If, for two neighboring spacepoints at equal radius $r=r_1$, the non-dilated distance between them is d_N , the dilated distance d_d is $d_d = d_N \cdot F_1(r_1)$.³ (The subscript N stands for "Newtonian" or "non-dilated", d stands for "dilated", and o stands for "observed")

For larger distances, the overall dilated path length following the spacepoints sp_1, \dots, sp_k is

$$pl_d(sp_1, \dots, sp_k) = \sum_{i=1}^{k-1} l.connection(sp_i, sp_{i+1}) \cdot F_1(r_1) \quad (3)$$

700 For paths on a (Schwarzschild metric) circumsphere, i.e., with constant radius r , this can be simplified
701 to

$$circumspherepl_d(sp_1, \dots, sp_k) = F_1(r) \cdot path_N(sp_1, \dots, sp_k)$$

by setting $path_N(sp_1, \dots, sp_k) = \sum_{i=1}^{k-1} l.connection(sp_i, sp_{i+1})$.

In Eq. (4), the dilated path length is obtained by multiplying the "non-dilated" path length by the factor F_1 , leaving aside that always $F_1 \leq 1$. In order to avoid misinterpretations and use a more meaningful base for the dilation factor, the dilation factor F_2 is introduced such that

$$circumspherepl_d(sp_1, \dots, sp_k) = F_2 \cdot path_N(sp_1, \dots, sp_k) \quad (4)$$

702 and F_2 is defined as $F_2(r) = F_1(r)/F_1(r_0)$, with r_0 being the minimal radius (e.g., $r_0 = 1$). This ensures
703 that F_2 is always $F_2 \geq 1$.

In radial direction, matters are more complicated because the factor F_2 varies with increasing radius. Let us define that instead of the factor F_2 , the dilated length in radial direction is dependent on a factor F_3

$$radialpl_d(sp_1, \dots, sp_k) = F_3 \cdot path_N(sp_1, \dots, sp_k). \quad (5)$$

704 The difference between F_3 and F_2 depends on several parameters. (For the special case of galactic
705 rotational curves, the parameters are described in Section 6.3.) In general, $F_3 < F_2$. This means that for
706 a sphere with radius r_d around a gravitational source the dilated circumference $c_d \neq 2\pi r_d$.

707 6.1.1. The observation of space distortion by a distant observer

708 For the analysis of the implications of the gravitational length dilations for cosmological models,
709 it is important to analyze the extent to which the length dilations can be observed by a distant observer,
710 such as an astronomer. The (3-dimensional) space distortion due to non-uniform length scale (like
711 other space curvature) can hardly be directly observed. It can be indirectly observed, by observing
712 an object's movement within a strong gravitational field or by observing the large-scale results of
713 dynamical space(-time) processes. Astronomers who observe the cosmos typically obtain projections of
714 3-dimensional curved space configurations to 2-dimensional images. Assuming that the Z-axis points
715 to the observer, the projections apply to the (X,Y)-plane. The length dilations in radial directions (from
716 the gravitational source) also appear in the projections; that is, they can be observed. As described
717 above, lengths in radial directions are dilated by the factor F_3 .

718 The dilation of lengths that are not purely in radial direction will also appear in the observations,
719 but only to the extent of the radial direction dilation. For example, according to the description given
720 above, the orbit around a gravitational source is dilated by the factor F_2 ($F_3 \leq F_2$). The distant observer,
721 however, will see the length of the orbit as $r_d \cdot 2\pi$, with the dilated radius $r_d = \text{non-dilated-radius} \cdot F_3$.

722 Furthermore, the length projections for an area of the cosmos (i.e., observations) must be seen in
723 relation to the space distortions in the surrounding space.

³ Under the assumption that the length dilation is not accomplished by changing the positions of the neighboring spacepoints.

724 6.2. *Non-smooth aggregation of spacetime dynamics*

725 In cosmology, it is well known that the strength of the gravitational field within a (dense)
 726 gravitational object increases in a different manner with increasing distance from the centre of mass
 727 than is the case outside the object. Section 4.2 explains that the process of aggregation of spacetime
 728 curvature changes resulting from several sources may be even more complex than assumed in the
 729 standard models. This may affect various aspects of the cosmological models.

730

731 The features of the proposed model of spacetime dynamics described above may have implications for
 732 many aspects of the present standard cosmological model. On the positive side, these features also
 733 offer opportunities for new explanations and interpretations in areas of cosmology that are not yet
 734 sufficiently understood. The major areas identified by the author in which the application of the model
 735 of spacetime dynamics may result in new explanations of cosmological observations are the following:

736 6.3. *Flat galaxy rotational curves and dark matter*

737 The existence of "dark matter" has been proposed as a possible explanation of the observed flat
 738 galaxy rotational curves, while an alternative explanation known as modified Newtonian dynamics
 739 (MOND) has also been proposed. Two further proposed theories explain the flat rotational curves by
 740 the existence of a new force: (i) a so-called entropic force (see [23]) or (ii) a so-called gravo-inductive
 741 field (see [24]).

742 Gravitational length dilation (see above) may provide yet another possible explanation for the flat
 743 galaxy rotational curves, in which the length and clock rate dilation (i.e. time dilation) yield a velocity
 744 larger than that deduced from the observed rotational curves. In simpler terms, the rotational curves
 745 are observed due to the spacetime curvature, in which the orbit has a larger length dilation factor than
 the radius.

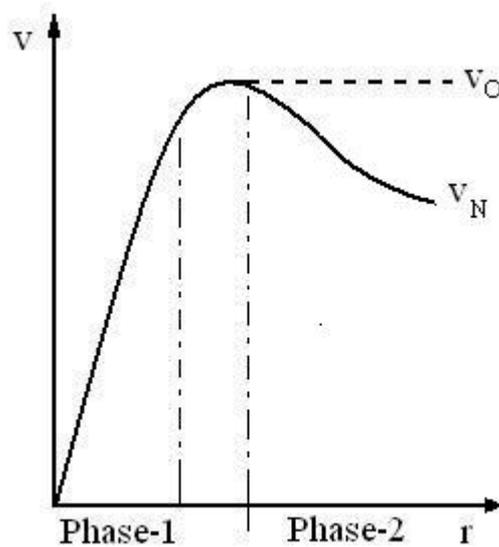


Figure 9. Velocities in galaxy rotational curves.

746

Let us consider a galaxy with rotational curves that are in accordance with (unmodified) Newtonian dynamics. The velocities of circular orbits are determined by Newton's force law $Ma = F$ for a test particle with mass M and acceleration a . For the rotational curves of stars of a galaxy, two

Table 2. Galaxy rotational curves

cases	dilation factors	circulation time, t	circumference c	radius r	velocity v
w/o dilation	1	t_N	$c_N = 2\pi r_N$	r_N	$v_N = c_N/t_N$
dilated	F_2, F_3	$t_d = t_N F_2$	$c_d = c_N F_2$	$r_d = r_N F_3$	$v_d = v_N$
observed	-	$t_o = t_N$	$c_o = 2\pi r_o$	$r_o = r_d = r_N F_3$	$v_o = c_o/t_o$
Example:					
w/o dilation	1	100	628	100	6.28
dilated	1.1, 1.05	110	690.8	105	6.28
observed	-	$t_o = t_N = 100$	$c_o = c_N = 660$	105	6.6

phases are distinguished with respect to the velocity v of circular orbits (see Fig. 9).⁴ In phase-1, when the star is within (or close to) the "bulge" that surrounds the center of the galaxy, the velocity according to Newtonian dynamics is

$$v(r) = \sqrt{\frac{GM(r)}{r}}. \quad (6)$$

G is the gravitation constant, r the radius, and $M(r)$ the mass, which is dependent on the radius. Resolving the dependency of the mass on the radius by application of the density law $M(r) = \rho_0 V$ (see [27]), results in $v(r)$ being proportional to r :

$$v(r) \sim r. \quad (7)$$

When the distance from the bulge is sufficiently large phase-2 applies where the velocity is expected to be

$$v(r) \sim \sqrt{\frac{1}{r}}. \quad (8)$$

747 For phase-1, the observations are in agreement with the expectation. Phase-2 presents a problem.
748 Instead of the decreasing velocity (Fig. 9, v_N), according to Eq. (9), a flat rotational curve is observed
749 (Fig. 9, v_o), i.e., the velocity observed is greater than expected.

750 In addition to the dark matter theory and the MOND theory, the length dilation assumed with
751 the proposed causal model of spacetime dynamics may provide another explanation for the flat galaxy
752 rotational curves. The explanation concerns, first of all, the differences in the length dilation of the
753 circumference of the rotational curve and the length dilation of the radius. This difference is the
754 difference between the factors F_3 and F_2 , described in Section 6.1. The difference between F_3 and F_2
755 causes differences between the observed values and the real values of the physical parameters as
756 described in Section 6.1.1.

757 In the context of galactic rotational curves, for the calculation of the value of F_3 , the following
758 points have to be taken into account:

- 759 1. In contrast to F_2 in Eq. (5), F_3 in Eq. (6) is the mean value for a path with varying radius r .
- 760 2. The complete range of radius to be considered includes the phase-1 part, the phase-2 part, and
761 the part between phase-1 and phase-2. In addition to the uncertainty as to where exactly phase-1
762 ends and where exactly phase-2 starts, the following points are difficult to quantify:
- 763 3. During phase-1, not only does F_2 (the basis for determining F_3) vary with r , but as indicated in
764 Eq. (7) the (effective) mass $M(r)$ also increases with increasing radius.
- 765 4. As described in Section 4.2, the author questions the general applicability of the density law
766 $M(r) = \rho_0 V$ for the determination of $M(r)$ for the complete phase-1.

⁴ Fig. 9 is only a schematic figure. No attempt has been made to show correct proportions.

Because of these points, the author is at present not able to provide a somewhat reliable calculation of F_3 for the observed flat galaxy rotational curves. At least it is possible to state a rough relation between F_3 and F_2 :

$$1 \leq F_3(r1) \leq F_2(r1). \quad (9)$$

767 In other words, F_3 for the path between radius $r=r0$ and radius $r=r1$ is greater than 1 and less than F_2
 768 for $r1$. This implies that for a circumference c_d with radius r_d , $c_d > 2\pi r_d$ – something that is possible
 769 only in curved space, and something that can never directly be observed by a distant observer.

770 The relationship between non-dilated entities, the dilated entities, and the observed entities with
 771 galaxy rotational curves is summarized in Table 2 with the three rows "w/o dilation", "dilated" and
 772 "observed". The essential table entry is the observed velocity, which is stated to be higher than the (real)
 773 dilated velocity (which is equal to the non-dilated velocity). The observed velocity v_o is $v_o = c_o/t_o$.
 774 c_o is measured or estimated by the observer in terms of the length (scale) of the observed radius r_o ,
 775 $c_o = 2\pi r_o$. The observed radius r_o , however, is roughly equal to the dilated radius, according to Section
 776 6.3. Because, for the observer, the circulation time t_o is equal to the non-dilated time t_N , the velocity v_o
 777 appears to be greater than the expected velocity v_N . In summary, the velocity v_o appears to be greater
 778 than the expected velocity v_N , because the length dilation effects are only partly visible to the distant
 779 observer.

780 6.4. Pioneer anomaly

781 A significant number of proposals have been published in an attempt to explain the Pioneer
 782 anomaly. [25] presents an excellent overview of the detailed nature of this anomaly and the efforts
 783 made to explain and study it. Among the possible explanations are the MOND-based explanation
 784 described above, "dark matter", and "gravitational forces due to unknown mass distributions and
 785 the Kuiper belt". Since 2012, the thermal recoil force has appeared to be the most widely accepted
 786 explanation (see [26]) for the Pioneer anomaly. As for the flat galaxy rotational curves (Section 6.1), the
 787 gravitational length dilation of the proposed model of spacetime dynamics provides a further possible
 788 explanation.

789 6.5. The expansion speed of the universe and dark energy

790 According to the standard model of cosmology, the universe is continuously expanding. The
 791 method used to determine the speed of this expansion is typically a measurement of the redshift of
 792 light emitted by the most distant stars. Based on these redshift measurements, astronomers have
 793 observed an increasing speed of expansion of the universe. The explanation currently favoured by
 794 astrophysicists for this increasing speed of expansion is a combination of several causes, with the
 795 largest contribution coming from "dark energy". All of the features of the model described above that
 796 can have an impact on cosmological models (i.e. (i) gravitational length dilation; (ii) the non-smooth
 797 aggregation of spacetime dynamics; and (iii) collective behaviour) may contribute to the varying (i.e.
 798 increasing) speed of expansion of our universe. Further work is required to obtain rough estimates of
 799 the possible contributions of these individual features and the combination of their effects.

800 7. Discussion

801 7.1. The special role of time

802 SRT and GRT have taught that space and time are integrated into spacetime. The major reason for
 803 taking this view is that in the laws and equations of SRT and GRT, time and space occur in combination,
 804 and the causal progression of the system state depends on the progression of the combination of both
 805 space and time. The causal model of spacetime dynamics presented in this article also implies a tight
 806 relationship of space and time, although with a different interpretation (see Section 3.1).

807 Nevertheless, there are also (good) reasons for not neglecting some fundamental differences
 808 between space and time. The major points where the concept of time assumed for the model described
 809 deviates from the time concept described (or implied) in some physics literature are:

810 • Arrow of time

811 The formal definition of a causal model (in general, not just for the model described in this
 812 article) assumes a constant direction in which time progresses, i.e., an arrow of time. Reverse
 813 progression of time or variable direction of time progression is just not supported by the model.
 814 The author believes that a causal model in general implies an arrow of time. In other words, a
 815 model that does not adhere to a unique constant direction of time would show more flexibility
 816 than nature shows in reality. The model would not be reality conformal.

817 • Time slices

818 With the goal of showing as much commonality as possible between space and time, some
 819 physics literature do not describe the extension of the time coordinate as differing from the
 820 extension of the space. In the formal definition of a causal model, the laws of physics that specify
 821 the state transitions can always access only the system state of the current point in time. It is not
 822 possible to access past or future time slices of system states. Models that would allow reference
 823 or even modifications of past or future system states are considered as (probably) not reality
 824 conformal and would be very complicated.

825 *7.2. Time dilation and/or length dilation?*

826 Both SRT and GRT predict, under specific circumstances, time dilation and/or length contraction.
 827 In textbooks covering SRT and GRT, it is not always clear whether (1) the two effects occur
 828 simultaneously, (2) the two effects are just two possible views from a non-local observer, or (3)
 829 there are cases where time dilation occurs (but no length contraction) and vice versa. For the proposed
 830 model of spacetime dynamics, length dilation is the primary effect. In the model, time dilation - more
 831 precisely, the clock rate dilation - is seen as a consequence of the length dilation. Length is a spatial
 832 attribute, while clock rate is a property of processes running in a causal subsystem. (In areas of space
 833 where there is no causal subsystem, there is no clock rate dilation, nor time dilation.) Despite the basic
 834 differences in the roles that time dilation and length dilation play (in the model), these functions are
 835 highly interrelated (see Section 3.1).

836 *7.3. The general dependency of the clock rate on the length scaling*

837 The model that assumes that GRT/SRT-based length dilations generally imply, as a secondary
 838 effect, a proportional clock rate increase/decrease for the process that executes in the length-dilated
 839 area of space requires a further non-trivial assumption. The additional rule is Conjecture 3.1 in Section
 840 3.1: "All physical processes can ultimately be broken down to length-related state changes, and changes
 841 in the length scaling, therefore, directly result in clock rate dilations of the affected process."
 842 If it were possible to identify a process that is not accompanied by some spatial state change (an
 843 example could be the decay of particles), and if it were possible to demonstrate that this process
 844 nevertheless adheres to GRT/SRT-predicted time dilation, this would prove that the model that
 845 assumes that time dilation is always a consequence of length dilation is wrong, or at least that it does
 846 not hold generally. The assumption that the rate of state change of a clock process and of arbitrary
 847 other processes that show a regular rate of state change depends in a predictable manner on the length
 848 scale of the space where the process executes is hard to believe. If the assumption could be confirmed,
 849 it would indicate another, even tighter relationship between time and space than is so far assumed
 850 with GRT.

851 **8. Conclusion**

852 The model of spacetime dynamics described in this article does not aim at providing another
 853 theory of the subject. Rather, it has the goal of providing a special model, namely a *causal model*, of the

854 subject for which a generally agreed upon theory exists. However, it is not possible to derive a causal
855 model of spacetime dynamics purely from GRT. GRT establishes a powerful base for the development
856 of the model, but supplementary statements and interpretations are required to construct a somewhat
857 complete (local) causal model of this area of physics. The described causal model is not claimed to
858 be the only possible or valid model of the subject. Alternative models, possibly focusing on specific
859 aspects, are imaginable. With those features of the model that could not be directly derived from
860 GRT and where, therefore, new solutions had to be invented, it may turn out that the solutions of the
861 present model have to be replaced by solutions that are in accordance with new experiments.

862 The two major items, where the proposed model deviates from the standard interpretations of
863 GRT and QFT are:

- 864 1. The assumption of the length dilation as the primary effect of space curvature that causes clock
865 rate dilation as a secondary effect.
- 866 2. The assignment of additional bosonic create operators for the out-connections of space points
867 (see Section 5).

868 Disregarding the uncertainties about the ultimate validity of certain details of the proposed model,
869 there are nevertheless a number of findings that the author believes are worth noticing:

- 870 • For an area of physics, it is mandatory that the construction of models of the complete dynamics
871 is feasible. The type of model that is best suited to describe the complete dynamics is the causal
872 model. The lack of feasibility of constructing a causal model of a theory of physics may be
873 considered as an indication of the incompleteness of the theory.
- 874 • As SRT and GRT show, space and time have to be viewed as integrated. The progression of time
875 can be described only in connection with spatial state changes. The length scaling within space
876 (including curvature) can only be described with reference to processes executing for a specific
877 time interval. However, besides this fundamental tight relation between space and time, it is also
878 necessary to point out the fundamental differences in the roles, structure, and properties of space
879 and time.

880 Further work is required to refine the model and make the ideas more solid. Dealing with discrete
881 space, time, and paths, refinements of the model may probably be achievable only with the help of
882 computer simulations.

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